Bell Work

Grades 8

2006 - 2007

Teacher Edition

Property of the Cleveland Municipal School District

Eugene T.W. Sanders, Ph.D., Chief Executive Officer
### Area Formulas

- **parallellogram**: \( A = bh \)
- **rectangle**: \( A = lw \)
- **trapezoid**: \( A = \frac{1}{2} h (b_1 + b_2) \)
- **triangle**: \( A = \frac{1}{2} bh \)

### Volume Formulas

- **cone**: \( V = \frac{1}{3} \pi r^2 h \)
- **cylinder**: \( V = \pi r^2 h \)
- **pyramid**: \( V = \frac{1}{3} Bh \) (\( B = \text{area of base} \))
- **right prism**: \( V = Bh \) (\( B = \text{area of base} \))

### Circle Formulas

- **Circumference**: \( C = 2\pi r \)
- **Area**: \( A = \pi r^2 \) \( \pi \approx 3.14 \) or \( \frac{22}{7} \)

### Sphere Formulas

- **Volume**: \( V = \frac{4}{3} \pi r^3 \)

### Distance Formula

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

**NOTE:** This formula sheet appears on the inside front and back covers of both the teacher and student additions of the bell work books. It is taken directly from the Ohio Department of Education and is the formula sheet given to students when they take the Grade 8 Mathematics Ohio Achievement Test.

Students should become comfortable with the reference sheet and know that they will have it to use on the OAT. They will have a similar reference sheet when they take the OGT, but it will contain more formulas.
Acknowledgements

Project Director: Julie Snipes-Rea
Copy Editor: Ben Snodgrass
Item Developers and Editors: Sally Mascia
Julie Snipes-Rea
Ben Snodgrass
Project Advisor: Dr. Donna Snodgrass

Special Thanks to:

Glenda Hathaway
Deputy Chief, Curriculum and Instruction

All of the CMSD teachers who have provided insightful comments and suggestions.
Cleveland Municipal School District
Mathematics 8th Grade Bell Work

⚠️ Purpose
The purpose of the 8th Grade Bell Work project is to give students the opportunity to continuously review math skills and concepts that they should have already learned—according to the Ohio Academic Content Standards. Students review these skills while practicing solving the types of problems they will see on the Ohio Achievement Tests.

The bell work materials provide a review of the 7th grade Ohio grade level indicators. In an effort to tie these review materials with regular classroom lessons, we have used the 8th grade math pacing calendar to organize the exercises. Since math concepts tend to build upon one another, the bell work exercises (previously covered material) can complement new material.

The other purposes of the bell work project are to provide a process to aid teachers in identifying skills in which their students need reinforcement and to provide teachers with tips and suggestions on understanding and teaching difficult math concepts.

⚠️ Materials
This teacher edition contains 130 bell work exercises.

Each bell work entry includes:
- The standard and grade level indicator assessed;
- The bell work activity (or activities);
- The scoring guide and/or answer key.

In addition, some bell work entries include Teacher Notes, which provide suggestions for reviewing important math content and skills with students.
How to Use the Materials

⚠ REVIEW the standard and grade level indicator. The teacher should identify and go over the indicator with the students before students begin each day's bell work. When students know and understand their learning targets, their sense of being in control of their learning increases and they are motivated to perform better.

⚠ STUDENTS COMPLETE the bell work activities. Teachers may either have the students work in small groups or individually on the bell work. Most of the activities are 1 or 2 open-ended items, although some days consist of several multiple choice questions. Most bell work should take just five minutes to complete.

⚠ GO OVER the bell work with students. It is highly recommended that students get immediate feedback on their bell work, if possible. The teacher may use a variety of methods for scoring—the teacher may have students score their own papers, have students exchange papers, or the teacher may score the papers. It is highly recommended that students are involved in the scoring and that teachers share scoring guides with the students so that the students are absolutely CLEAR about what a full response looks like. In many cases, items will be “recycled” and students will have a second opportunity to attempt a similar item.

⚠ REINFORCE or re-teach specific skills when the students demonstrate a need. When necessary, use the Teacher Notes and your instructional strategies to reinforce or re-teach the skills assessed in the bell work activity.

Final Note: The Order of Bell Work Entries

Students are cycled through the same indicator multiple times throughout the book. However, a teacher may choose to utilize a bell work entry that assesses a specific GLI on a specific day. For example, a teacher may choose to use a particular bell work entry to assess prior knowledge for a new skill they are planning to teach. If a teacher chooses to do this, the bell work entries are listed by GLI in the Table of Contents.
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Exercise #1

Standard: Number, Number Sense and Operations

Indicator: NNSO#2 (7th Grade)

Explain the meaning of exponents that are negative or 0.

1. Which of the following is equivalent to $10^{-4}$?
   - a. 0.0001
   - b. 0.0004
   - c. -10,000
   - d. -0.00001

2. Which of the following is equivalent to $10^0$?
   - a. 10
   - b. 0.1
   - c. 1
   - d. 100

3. Write each of the following as a fraction or whole number without exponents. Show how you got your answer.

   $6^{-1}$
   
   $4^{-2}$
   
   $8^0 \times 2^{-3}$
Answer Key

Point Value: 5 Points (1 point each)

1. A
2. C
3. $\frac{1}{6}$
4. $\frac{1}{16}$
5. $\frac{1}{8}$
Exercise #2

Standard: **Number, Number Sense and Operations**

Indicator: **NNSO#2 (7th Grade)**

*Explain the meaning of exponents that are negative or 0.*

Matt’s teacher said that any number to the power of 0 equals 1. Matt could not understand this rule. His classmate Arisha said that she could show him that $10^0 = 1$. She wrote:

**Step 1:**

$10^5 \div 10 = 10^4$

$10^4 \div 10 = 10^3$

$10^3 \div 10 = 10^2$

$10^2 \div 10 = 10^1$

$10^1 \div 10 = 10^0$

**Step 2:**

$10^1 \div 10 = 10 \div 10$

**Step 3:**

$10 \div 10 = 1$

Therefore, $10^0 = 1$

Is Arisha’s logic correct? Support your answer by explaining or proving whether each of Arisha’s steps is correct or not.
Scoring Guide

Point Value: 4 Points

4 Points: The response includes the following 4 elements:

- A statement that her logic is correct.
- Correct explanation for step 1 that says if you divide 10 to the power of x by 10, you get 10 to the power of x-1; OR when dividing two numbers with exponents, if the base numbers are the same, subtract the exponents; OR the student may do calculations to show that the equations are correct.
- Correct explanation for step 2 that says that any number to the power of 1 is that original number;
- Correct explanation for step 3 that says that any number divided by itself (except 0) is 1.

3 Points: The response includes 3 of the elements of a 4-point response.

2 Points: The response includes 2 of the elements of a 4-point response.

1 Point: The response includes 1 of the elements of a 4-point response.

0 Points: The response shows no understanding of the concept.

Teacher Note: The intention of this problem is to help students explore the rules of exponents by examining patterns. Students should determine that there is a pattern to the values in step 1 and work backwards to determine the value of the number raised to a power of zero. For this example, when using the powers of ten, the value is divided by ten each time the exponent is reduced by one.
Exercise #3

Evaluate the following expressions. Write your answers in exponent form. Show how you got your answers.

a. \(10^4 \div 10^2\)

b. \(5^2 \cdot 5^3\)

c. \(2^5 \cdot 2^6\)

d. \(7^3 \div 7^4\)
Answer Key
Point Value: 4 Points (1 point each)

a. $10^2$

b. $5^5$

c. $2^{11}$

d. $7^{-1}$
Exercise #4

Standard: **Number, Number Sense and Operations**

Indicator: NNSO#3 (7th Grade)
**Describe differences between rational and irrational numbers**

1. Which of the following sets of numbers does \( \sqrt{7} \) belong to?
   - a. Irrational Numbers
   - b. Whole Numbers
   - c. Rational Numbers
   - d. Integers

2. Which of the following sets does \( \frac{1}{9} \) belong to?
   - a. Irrational Numbers
   - b. Whole Numbers
   - c. Rational Numbers
   - d. Integers

3. Which of the following statements is true about rational numbers?
   - a. All rational numbers are integers.
   - b. Integers are not rational numbers.
   - c. Some whole numbers are not rational numbers.
   - d. All integers are rational numbers.
Answer Key

Point Value: 3 Points (1 point each)

1. A
2. C
3. D

Teacher Notes:

Students should know that there are different sets and subsets of numbers.

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<th>Classification</th>
<th>Examples</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting Numbers/Natural Numbers</td>
<td>1, 12, 332, 999</td>
<td>Numbers used to count</td>
</tr>
<tr>
<td>Whole Numbers</td>
<td>0, 1, 2, 3…</td>
<td>Zero and all counting numbers</td>
</tr>
<tr>
<td>Integers</td>
<td>-23, -11, 0, 1, 44</td>
<td>Zero, whole numbers (positive), and their opposites</td>
</tr>
<tr>
<td>Rational Numbers</td>
<td>$\frac{3}{4}$, -0.45, 3, 5.14</td>
<td>Any number that can be written in the fraction form $\frac{a}{b}$ where a and b are integers and b is not equal to zero; when in decimal form, the number terminates or repeats.</td>
</tr>
<tr>
<td>Irrational Numbers</td>
<td>3.14159… (pi), $\sqrt{5}$</td>
<td>Any number that CANNOT be written in the fraction form $\frac{a}{b}$ where a and b are integers and b is not equal to zero; when in decimal form, the number does not terminate or repeat.</td>
</tr>
</tbody>
</table>

Continued on next page →
The hierarchy of these numbers looks something like this:

Rational numbers are simply numbers that can be written as fractions or ratios (this tells you where the term rational comes from). A rational number is any number that can be expressed in the form \( \frac{a}{b} \) or \( \frac{\hat{a}}{\hat{b}} \), where \( a \) and \( b \) are integers and \( b \neq 0 \). They are called rational because they are the ratio, or quotient, of two integers. Rational numbers include integers as well as positive and negative fractions. Each rational number can be associated with a point on a number line.

Rational numbers, when written in decimal form are terminating decimals or repeating decimals.

Fractions, repeating and terminating decimals, and percents are all different forms that can be used to write the same rational number:

- An integer can be written as a fraction simply by giving it a denominator of one, so any integer is a rational number.
  \[ 6 = \frac{6}{1}; \quad 4 = \frac{4}{1}; \quad 0 = \frac{0}{1} \]

- A terminating decimal can be written as a fraction simply by writing it the way you say it: \( 3.75 = \text{three and seventy-five hundredths} = \frac{375}{100} \), then adding if needed to produce a fraction:
  \[ \frac{300}{100} + \frac{75}{100} = \frac{375}{100} \]. So, any terminating decimal is a rational number.

- A repeating decimal can be written as a fraction using algebraic methods, so any repeating decimal is a rational number. (e.g. \( 1/9 = 0.111... \), \( 2/9 = 0.222... \), etc.)

- A percent can be written as a fraction simply by writing the percent over 100 (e.g. \( 40\% = 40/100 \)).
Exercise #5

<table>
<thead>
<tr>
<th>Number</th>
<th>Rational or Irrational?</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$7^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6314...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.333</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard: **Number, Number Sense and Operations**

Indicator: **NNSO#3 (7th Grade)**

Describe differences between rational and irrational numbers.

Label each number as rational or irrational. Justify how you know which type of number it is.
Answer Key
Point Value: 6 Points (1 point each)

<table>
<thead>
<tr>
<th>Number</th>
<th>Rational or Irrational?</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Rational</td>
<td>The number 3 is an integer; OR The number 3 can be written as a fraction ( \frac{3}{1} ); OR The number 3 can be written as a terminating decimal (3.0).</td>
</tr>
<tr>
<td>12%</td>
<td>Rational</td>
<td>Can be written as a fraction, ( \frac{12}{100} )</td>
</tr>
<tr>
<td>(7^2)</td>
<td>Rational</td>
<td>Integer OR can be written as a fraction, ( \frac{49}{1} )</td>
</tr>
<tr>
<td>0.6314...</td>
<td>Irrational</td>
<td>Is a decimal that does not repeat or terminate</td>
</tr>
<tr>
<td>0.333</td>
<td>Rational</td>
<td>Is a decimal that repeats</td>
</tr>
<tr>
<td>(\pi)</td>
<td>Irrational</td>
<td>Has a decimal equivalent that does not repeat or terminate</td>
</tr>
</tbody>
</table>
Exercise #6

Standard: Number, Number Sense and Operations

Indicator: NNSO#1 (7th Grade)
Demonstrate an understanding of place value using powers of 10 and write large numbers in scientific notation.

1. What digit is in the hundredths place in the number 7450.123?

   a. 1
   b. 2
   c. 3
   d. 4

2. What is the place value of the 3 in the number 749.325?

   a. ones
   b. tens
   c. tenths
   d. hundredths

3. What is the place value of the 4 in the number 749.235?

   ____________________________

4. What is the place value of 3 in the number 315.478?

   ____________________________
Answer Key

Point Value: 4 Points (1 point each)

1. B
2. C
3. tens
4. hundreds
Exercise #7

1. Deimos, a moon of the planet Mars, takes approximately 1860 minutes to orbit the planet. How is this number represented in expanded form?
   - a. \((1 \times 1000) + (8 \times 100) + (6 \times 10) + (0 \times 1)\)
   - b. \((1 \times 1000) \times (8 \times 100) \times (6 \times 10) \times (0 \times 1)\)
   - c. \((1 + 1000) + (8 + 100) + (6 + 10) + (0 + 1)\)
   - d. \((1 + 1000) \times (8 + 100) \times (6 + 10) \times (0 + 1)\)

2. The diameter of the planet Earth is 12,756 kilometers. Write this number in expanded form.

   

3. The diameter of Pluto is about 2274 kilometers. How is this number represented in exponential form?
   - a. \((2 \times 10^4) + (2 \times 10^3) + (7 \times 10^2) + (4 \times 10^1)\)
   - b. \((2 \times 10^3) \times (2 \times 10^2) \times (7 \times 10^1) \times (4 \times 10^0)\)
   - c. \((2 \times 10^3) + (2 \times 10^2) + (7 \times 10^1) + (4 \times 10^0)\)
   - d. \((2 + 10^4) \times (2 + 10^3) \times (7 + 10^2) \times (4 + 10^1)\)

4. The population of the city of Cleveland in the 2000 census was 478,403. How is this number represented in exponential form?
Answer Key
Point Value: 4 Points (1 point each)

1. A
2. \((1 \times 10,000) + (2 \times 1000) + (7 \times 100) + (5 \times 10) + (6 \times 1)\)
3. C
4. \((4 \times 10^5) + (7 \times 10^4) + (8 \times 10^3) + (4 \times 10^2) + (0 \times 10^1) + (3 \times 10^0)\)

**Teacher Note:** Students need to know that a number can be written in a variety of forms. Here is an example using the number 325,469.

<table>
<thead>
<tr>
<th>Form Type</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Form</td>
<td>325,469</td>
</tr>
<tr>
<td>Expanded Form</td>
<td>((3 \times 100,000) + (2 \times 10,000) + (5 \times 1,000) + (4 \times 100) + (6 \times 10) + (9 \times 1))</td>
</tr>
<tr>
<td>Exponential Form</td>
<td>((3 \times 10^5) + (2 \times 10^4) + (5 \times 10^3) + (4 \times 10^2) + (6 \times 10^1) + (9 \times 10^0))</td>
</tr>
<tr>
<td>Scientific Notation</td>
<td>(3.25469 \times 10^5)</td>
</tr>
</tbody>
</table>

You may want to have students write the same numbers in a variety of forms.
Exercise #8

Standard: Number, Number Sense and Operations
Indicator: NNSO #8 (7th Grade)
Develop and analyze algorithms for computing with percents and integers, and demonstrates fluency in their use.

James went to the store to get a new pair of basketball shoes. He chose a pair of shoes that cost $42.00. The salesperson told James that all basketball shoes were on sale for 30% off of the original price. James thought that the shoes would cost $12.60, but the cashier told him that he was incorrect. What mistake did James make? Show how to calculate the sale price of the shoes.
Scoring Guide
Total Points: 2

2 Points: Student response indicates that James calculated 30% of the original price, not 30% off of the original price. Student shows correct calculations for finding the sale price of $29.40.

1 Point: Student response is missing either of the following: (1) correct explanation of James’s mistake or (2) the correct calculations for finding the sale price.

0 Points: No response, or both portions of the problem are incorrect.
Exercise #9

Standard: Number, Number Sense and Operations

Indicator: NNSO#9 (7th Grade)
Represent and solve problem situations that can be modeled by and solved using concepts of absolute value, exponents, and square roots (for perfect squares).

1. $|s| = 9$, $s$ can equal the integers _____ or _____.

2. $|-6| + 17 = \underline{\hspace{2cm}}$

3. $|-6 + 17| = \underline{\hspace{2cm}}$

4. On the number line below, show all of the solutions to the following equation.

$$|n| = 5$$
Scoring Guide

Point Value: 6 Points (2,1,1,2)

1. $s = -9$ or 9  (1 point for each correct answer)
2. $|\text{-6}| + 17 = 23$  (1 point for correct response)
3. $|\text{-6} + 17| = 11$  (1 point for correct response)
4. Student marks both -5 and +5 as solutions for $n$. (1 point each)
Exercise #10

Standard: Number, Number Sense and Operations

Indicator: NNSO#9 (7th Grade)
Represent and solve problem situations that can be modeled by and solved using concepts of absolute value, exponents, and square roots (for perfect squares).

1. What values can you substitute for \( k \) so that the following number sentence is true?

\[ 3 \cdot |k| = 12 \]

2. Chanel went on an all-day hike in the mountains. She started her hike at an elevation of 2,700 feet above sea level. During each section of the hike, she kept track of her change in elevation.

<table>
<thead>
<tr>
<th>Ascent and Descent during Chanel's Hike</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section 1</td>
</tr>
<tr>
<td>Elevation Change (ft.)</td>
</tr>
</tbody>
</table>

A. At the end of her hike, what is Chanel's elevation?
B. During her day of hiking, how many feet did she ascend and how many feet did she descend?
Scoring Guide

Point Value: 4 (2 + 2)

Question 1

2 Points: Response shows that $k = -4$ or $+4$.

1 Point: Response includes one of the two possible solutions for $k$. OR
         Student makes an arithmetic error, but the two solutions given are a positive number and its corresponding negative number.

0 Points: Response does not demonstrate an understanding of the concept.

Question 2

2 Points: Response indicates that:
   
   • Chanel’s elevation at the end of the day is 3,925 feet above sea level. (It is also acceptable if the response states that her elevation is 1225 feet higher than it was at the beginning of the hike.);
   
   • Chanel ascended 2350 feet and descended a total of 1125 feet.

1 Point: Student responds correctly to one of the two parts of the prompt.

0 Points: Response does not demonstrate an understanding of the concept.
Exercise #11

Standard: Number, Number Sense and Operations

Indicator: NNSO#9 (7th Grade)
Represent and solve problem situations that can be modeled by and solved using concepts of absolute value, exponents, and square roots (for perfect squares).

1. A square table has an area of 625 square inches. Find the length of a side of the table. Show your calculations.

2. Joanna won a lottery and was given two options for her prize.

   Option 1: Receive $1000 today.

   Option 2: Receive $1 today. For the next 6 days, receive triple the amount you received the previous day.

   In order to get the most money, which option should Joanna choose? Show calculations to justify your answer.
Scoring Guide

Point Value: 4 (2+2 points)

Question 1:
2 Points: Student shows calculations and arrives at the correct answer: the side length is 25 inches.
1 Point: Student gives the correct answer but does not show calculations. OR Student shows correct process but makes an arithmetic error.
0 Points: No response. OR Response does not reflect understanding of the concept.

Question 2:
2 Points: Student argues that Joanna should choose Option 2. With Option 2, she will receive $3^0 + 3^1 + 3^2 + 3^3 + 3^4 + 3^5 + 3^6 = $1093.
1 Point: Student argues that Joanna should choose Option 2, but does not provide justification. OR Student makes minor error that still shows an understanding of using exponents, e.g. only calculates $3^6 = 729$ and argues for Option 1.
0 Points: No response. OR Response does not reflect an understanding of the concept.
Exercise #12

Standard: Patterns, Function and Algebra

Indicator: PFA #1 (7th Grade)
Represent and analyze patterns, rules and functions with words, tables, graphs and simple variable expressions.

The mass of a box is 150 grams when it is empty. Inside the box are cubes that each have a mass of 20 grams.

a. Make a table of values for the total mass of the box and cubes if it contains 1 cube, 2 cubes, 3 cubes, 4 cubes, and 5 cubes.

b. Write an equation for the relationship between the number of cubes, \( n \), and the total mass of the box and cubes, \( T \).

c. Describe in words how to find the mass of the box when it contains 55 cubes and give the total mass.
Scoring Guide

Point Value: 4 Points

4-points Response indicates:

- A table of values correctly constructed with the first column showing the number of cubes and the second column showing the total mass of the box and cubes.
- The correct values are given for the total mass of the box and cubes. For example:

<table>
<thead>
<tr>
<th>Number of Cubes</th>
<th>Total Mass of the Box and Cubes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>150</td>
</tr>
<tr>
<td>1</td>
<td>170</td>
</tr>
<tr>
<td>2</td>
<td>190</td>
</tr>
<tr>
<td>3</td>
<td>210</td>
</tr>
<tr>
<td>4</td>
<td>230</td>
</tr>
<tr>
<td>5</td>
<td>250</td>
</tr>
</tbody>
</table>

- The total mass of the box $T$, and $n$ cubes written as an equation. For example: $T = 150 + 20n$
- Describes in words how to find the total mass of the box and gives the total mass as 1250 grams. For example: The total mass of the box with 55 cubes is 1250 grams. You would multiply 55 cubes times 20 grams and then add 150 grams.

3-points Response contains all elements of a 4 point response with an error in ONE of the bulleted score points.

2-points Response contains all elements of a 2 point response with an error in TWO of the bulleted score points.

1-point Response contains all elements of a 2 point response with an error in THREE of the bulleted score points.

0-points No response OR response contains FOUR or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #13

Standard: Patterns, Function and Algebra

Indicator: PFA #1 (7th Grade)
Represent and analyze patterns, rules and functions with words, tables, graphs and simple variable expressions.

The mass of a box is 120 grams when it is empty. Inside the box are cubes that each have a mass of 15 grams. The equation for finding the total mass of the box containing any number of cubes is $T = 120 + 15n$, with $T$ representing the total mass of the box and cubes, and $n$ representing the number of cubes. Create a graph of the equation for any number of cubes and the corresponding total mass of the box and cubes. Be sure to label your axes and create an appropriate title.
Scoring Guide

Point Value: 4 Points

4-points: Response indicates the equation correctly graphed showing:

- The x-axis labeled with the number of cubes (the independent variable) and using appropriate, consistently progressing intervals;
- The y-axis labeled with the total mass in grams (the dependent variable) and using appropriate, consistently progressing intervals;
- The y-intercept at point (0, 120);
- The slope of the straight line rising positively at 15/1 and passing through point (4, 180);

For example:

NOTE: It is not necessary for students to have graphed ten (10) cubes; however, they should have graphed a sufficient number so as to show the function clearly.

NOTE: Students may have used smaller intervals than 20 grams for the total mass.
3-points: Response contains all elements of a 4 point response with an error in ONE of the bulleted score points.

2-points: Response contains all elements of a 2 point response with an error in TWO of the bulleted score points.

1-point: Response contains all elements of a 2 point response with an error in THREE of the bulleted score points.

0-points: No response OR response contains FOUR or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #14

Standard: Patterns, Function and Algebra

Indicator: PFA #2 (7th Grade)
Generalize patterns by describing in words how to find the next term.

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Shape Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

a. Look at the table above. If the sequence continues, how many squares are in 
the 4th diagram? ________________

b. Describe in words how you found it.
Scoring Guide

Point Value: 2 Points

2-points Response indicates:

- There are 10 squares in the 4th term.
- Describes in words how the 4th term was found. (For example, multiply the term number by 2 and then add 2; or add 2 to the previous number of squares; etc.)

1-point Response contains all elements of a 2 point response with an error in ONE of the bulleted score points.

0-points No response OR response contains TWO or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #15

Standard: Patterns, Function and Algebra

Indicator: PFA #3 (7th Grade)
Recognize and explain when numerical patterns are linear or nonlinear progressions; e.g., 1, 3, 5, 7 ... is linear and 1, 3, 4, 8, 16 ... is nonlinear.

Anthony bought a car valued at $40,000. Every year it decreased in value. The first year it decreased in value $6000. The second year it decreased in value another $5800. The third year it decreased in value an additional $5600. This pattern continued.

a. Make a table to find the value of the car each year up to the fifth year.
b. Does the value of the car each year represent a linear pattern or a nonlinear pattern? Explain your answer.
Scoring Guidelines

Point Value: 2 Points

2-points Response indicates:

- A table that shows the value of the car in the 5<sup>th</sup> year. For example:

<table>
<thead>
<tr>
<th>Number of Years</th>
<th>Value of the car</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>40,000</td>
</tr>
<tr>
<td>1</td>
<td>34,000</td>
</tr>
<tr>
<td>2</td>
<td>28,200</td>
</tr>
<tr>
<td>3</td>
<td>22,600</td>
</tr>
<tr>
<td>4</td>
<td>17,200</td>
</tr>
<tr>
<td>5</td>
<td>12,000</td>
</tr>
</tbody>
</table>

- Explains that this is a nonlinear pattern AND gives an explanation of their reasoning. Examples of explanations include but are not limited to: because the value of the car decreases by a different amount each year; because the value of the car does not decrease at a constant rate; because if the values are graphed they do not form a straight line; etc.

1-point Response contains all elements of a 2 point response with an error in ONE of the bulleted score points.

0-points No response OR response contains TWO or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #16

Standard: Patterns, Function and Algebra

Indicator: PFA #3 (7th Grade)
Recognize and explain when numerical patterns are linear or nonlinear progressions; e.g., 1, 3, 5, 7 ... is linear and 1, 3, 4, 8, 16 ... is nonlinear.

<table>
<thead>
<tr>
<th>Term number</th>
<th>Shape sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>:</td>
</tr>
<tr>
<td>2</td>
<td>::</td>
</tr>
<tr>
<td>3</td>
<td>:: ::</td>
</tr>
</tbody>
</table>

a. How many dots would be in the eighth figure of the table above?
b. Does this sequence represent a linear or a nonlinear pattern? Explain your answer.
Scoring Guide

Point Value: 2 Points

2-points Response indicates:

- Indicates that the eighth figure would have 64 dots.
- Explains that this is a nonlinear pattern AND gives an explanation.
  Examples of explanations include but are not limited to: the pattern is
  exponential OR the pattern does not make a straight line when
  graphed OR the pattern does not increase at a constant rate, etc.

1-point Response contains all elements of a 2 point response with an error in
ONE of the bulleted score points.

0-points No response OR response contains TWO or more errors in the bulleted
score points OR demonstrates no understanding of concept.
**Exercise #17**

Standard: Patterns, Function and Algebra

Indicator: PFA #4 (7th Grade)

Create visual representation of equation-solving processes that model the use of inverse operations.

Andrea used the following number lines to show how she solved the equation $x + 5 = 7$.

Her explanation is as follows:

$x + 5 = 7$

Because $x + 5$ is equal to 7, the brace for $x + 5$ must be the same length as 7 intervals.

$x + 5 - 5 = 7 - 5$

To find the value of $x$, you use the inverse of adding 5, which is subtracting 5 from both sides of the equation. This is shown by the arrow going to the left 5 intervals from 7.

$x = 2$

This gives you $7 - 5$ which is 2. Therefore, $x = 2$.

1. Using Andrea’s explanation, which of the following number lines shows a way to solve the equation $x + 7 = 12$?

   ![Number Line Options]

   ... continued on the next page.
2. Using Andrea’s explanation as a guide, explain why the answer you chose is the correct model for solving the equation $x + 7 = 12$. 
Scoring Guide

Point Value: 3 Points (1 point + 2 points)

1. 0 b.

2. 2 points  
   Response indicates:
   
   - The explanation that \( x + 7 \) is the same as 12, so the brace for \( x + 7 \) must be the same length as 12 units.
   - To find the value of \( x \), you subtract 7 units from 12. The arrow goes backward (to the left) on the number line 7 units to find the amount represented by \( x \), which is 5. This would give you \( 5 + 7 = 12 \) as the correct representation.

1 point  
Response contains all elements of a 2 point response with an error in ONE of the bulleted score points.

0 points  
No response OR response contains TWO or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #18

Standard:  Patterns, Function and Algebra
Indicator:  PFA #4 (7th Grade)
Create visual representation of equation-solving processes that model the use of inverse operations.

When Jacob solved the equation $2x - 3 = 5$, he wrote down his solution as $x = 4$. Derek said that answer was wrong, and that $x = 16$. One of the students is correct and one is incorrect. Which student had the correct answer? Explain why the other student’s answer was incorrect.
Scoring Guide

Point Value: 2 Points

2-points Response indicates:

- Jacob has the correct answer.
- An explanation that tells why Derek’s answer is incorrect. (For example, because although Derek correctly added 3, he then multiplied by 2 (when he should have divided by 2); OR Derek’s answer is incorrect because if you replace 4 for \(x\) in the equation and then solve, you get \(2 \cdot 4 - 3 = 5\), which is \(8 - 3 = 5\). Since this is Jacob’s answer, Derek must be wrong; OR Derek’s answer is incorrect because if you replace 16 for \(x\) in the equation and then solve, you get \(2 \cdot 16 - 3 \neq 5\), which is \(32 - 3 = 29\), etc.)

1-point Response contains all elements of a 2 point response with an error in ONE of the bulleted score points.

0-points No response OR response contains TWO or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #19

Standard: **Patterns, Function and Algebra**

Indicator: PFA #4 (7th Grade)
Create visual representation of equation-solving processes that model the use of inverse operations.

Sarah used Algebra Tiles to solve the equation $2x + (-3) = -5$. Her solution is pictured below.

<table>
<thead>
<tr>
<th>Original Equation</th>
<th>Step One</th>
<th>Step Two</th>
<th>Step Three</th>
</tr>
</thead>
</table>

a. What is Sarah’s solution to the equation?
b. What did Sarah do in Step One?
c. What did Sarah do in Step Two?
d. What did Sarah do in Step Three?

**HINT:** Remember that ✠ represents the variable $x$, □ represents a positive unit amount; and ■ represents a negative unit amount.
Scoring Guide

Point Value: 4 Points

4-points Response indicates:

- The solution is $x = -1$.
- Sarah used the additive inverse in Step One. (For example, student might explain that Sarah used the inverse of $-3$, which is $+3$ on both sides of the equation, OR added $+3$ to both sides of the equation, OR that they created the equation $2x + (-3) + 3 = -5 + 3$, etc.)
- Sarah simplified the equation in Step Two. (For example, student might explain that Sarah eliminated zero pairs to simplify the equation, OR that they simplified the equation to $2x = -2$, etc.)
- Sarah used the multiplicative inverse in Step Three. (For example, student might explain that Sarah divided both sides of the equation by 2, OR that they multiplied both sides of the equation by $\frac{1}{4}$, OR that each $x$ corresponds to $-1$, etc.)

3-point Response contains all elements of a 4 point response with an error in ONE of the bulleted score points.

2-point Response contains all elements of a 4 point response with an error in TWO of the bulleted score points.

1-point Response contains all elements of a 4 point response with an error in THREE of the bulleted score points.

0-points No response OR response contains FOUR or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #20

Standard: Patterns, Function and Algebra

Indicator: PFA #4 (7th Grade)
Create visual representation of equation-solving processes that model the use of inverse operations.

Shondra used Algebra Tiles to solve an equation. Her solution is pictured below.

a. What was the original equation?

b. What is her solution?

c. Explain her solution using words, numbers and/or symbols.

HINT: Remember that □ represents a variable, ■ represents a negative unit amount, and □ represents a positive unit amount.)
Scoring Guide

Point Value: 3 Points

3-points Response indicates:

- The original equation is \(-2 = x - 5\) (or an equivalent equation).
- The solution is \(x = 3\).
- An explanation that Shondra first added 5 to both sides of the equation and then eliminated Zero Pairs.

2-points Response contains all elements of a 3 point response with an error in ONE of the bulleted score points.

1-points Response contains all elements of a 3 point response with an error in TWO of the bulleted score points.

0-points No response OR response contains THREE or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #21

Standard: Measurement
Indicator: Measurement #9 (Grade 7)
Describe what happens to the surface area and volume of a three-dimensional object when the measurements of the object are changed.

A regular can of tomato soup has a diameter of 4 cm and a height of 8 cm. The company also makes cans of tomato soup in Jumbo size. A Jumbo can of tomato soup has a diameter of 8 cm and a height of 16 cm. For the following questions use $\pi=3.14$.

a. How much more soup will the Jumbo can hold than will the regular can? Use numbers, words, or symbols to show your work.

b. Soup labels cover only the sides of the cans. What is the difference in the sizes of the labels? Use numbers, words, or symbols to show your work.
Scoring Guide

Point Value: 8 Points (4 points each)

Part A:
4 Points: A complete response contains the following.
- Student finds the volume of the Regular sized can.
  Regular: \( \pi r^2 h = 32\pi \text{ cm}^3 = 100.5 \text{ cm}^3 \)
- Student finds the volume of the Jumbo sized can.
  Jumbo: \( \pi r^2 h = 256\pi \text{ cm}^3 = 803.8 \text{ cm}^3 \)
- Student calculates the difference in volume.
  \( 803.8 \text{ cm}^3 - 100.5 \text{ cm}^3 = 703.3 \text{ cm}^3 \)
- Student gives answer that the Jumbo can holds \( 224\pi \text{ cm}^3 \) or 703.3 cm³ more than does a Regular can of soup.

3 Points: Student response contains 3 of the 4 necessary attributes listed above.
2 Points: Student response contains 2 of the 4 necessary attributes listed above.
1 Point: Student response contains 1 of the 4 necessary attributes listed above.
0 Points: Student response contains none of the necessary attributes listed above.

Part B:
4 Points: A complete response contains the following.
- Student finds the area of the Regular label.
  Regular: \( 2\pi rh = 32\pi \text{ cm}^2 = 100.5 \text{ cm}^2 \)
- Student finds the area of the Jumbo label.
  Jumbo: \( 2\pi rh = 128\pi \text{ cm}^2 = 401.9 \text{ cm}^2 \)
- Student finds the difference in area of the labels.
  \( 401.9 \text{ cm}^2 - 100.5 \text{ cm}^2 = 301.4 \text{ cm}^2 \)
- Student give the answer that the Jumbo label will be \( 96\pi \text{ cm}^2 \) or 301.4 cm² larger than the Regular label.

3 Points: Student response contains 3 of the 4 necessary attributes listed above.
2 Points: Student response contains 2 of the 4 necessary attributes listed above.
1 Point: Student response contains 1 of the 4 necessary attributes listed above.
0 Points: Student response contains none of the necessary attributes listed above.
Exercise #22

1. What is the ratio of the measure of $AB$ to the measure of $DE$?

   - a. 1:2
   - b. 2:1
   - c. 1:1
   - d. 1:½

2. What is the ratio of the measure of $\angle ABC$ to the measure of $\angle EFD$?

   - a. 1:2
   - b. 2:1
   - c. 1:1
   - d. 1:½
Answer Key

Point Value: 2 (1 point each)

1. A. 1:2

2. C. 1:1

Teacher Note: When dealing with similar triangles, it is important for students to remember that the corresponding angles are *always* congruent and that they have a ratio of 1 to 1. When looking at side lengths, they have to look more closely; however, the ratios for all of the sides are the same in any given pair of similar triangles.
Exercise #23

Standard: Geometry and Spatial Sense

Indicator: Geometry #1 (Grade 7)
Use proportional reasoning to describe and express relationships between parts and attributes of similar and congruent figures.

Jim and Theresa are having a disagreement about similar triangles $FGH$ and $JKL$. Jim thinks that the measure of $\angle KJL$ is $48^\circ$. Theresa thinks that the measure of $\angle KJL$ is $24^\circ$. Which student has the correct answer? Explain how you know that the answer is correct.
Scoring Guide

Point Value: 2

2 Points: Response contains the following information.

- Theresa is correct—the measure of \( \angle KJL \) is 24°.
- Response justifies why the measure of \( \angle KJL \) is 24°. For example, since the triangles are similar, angles that correspond have the same measure.

1 Point: Response reflects one of the two bulleted points.

0 Points: Response does not contain either of the two parts of the 2-point response.

**Teacher Note:** It is important that students do not confuse the ratio of sides of similar triangles with the ratio of angles. The ratio of corresponding angles of similar triangles is always 1 to 1. Triangles with different angle measures cannot be similar.
Exercise #24

Standard: Geometry and Spatial Sense
Indicator: Geometry #4 (Grade 7)
Determine necessary conditions for congruence of triangles.

With the given information, it is not possible to prove that the triangles in Group 1 are congruent. However, it is possible to prove that the triangles in Group 2 are congruent.

a. What property allows you to prove that the triangles in Group 2 are congruent?

b. What more would you need to know in order to say that the triangles in Group 1 are congruent?
Scoring Guide

Point Value: 2

2 Points: Response includes the following:

- Explanation the Side-Angle-Side rule allows one to say that the triangles in Group 2 are congruent.
- Description of what information would allow one to say that the triangles in Group 1 are congruent, such as one of the following:
  - $\angle ABC \cong \angle DEF$
  - $\overline{AC} \cong \overline{DF}$
  - $\angle BAC \cong \angle EDF$ and $\angle ABC \cong \angle DEF$

1 Point: Response contains 1 of the 2 parts of the complete response.

0 Points: Response contains 0 of the 2 parts of the complete response.
**Teacher Note:** Students may need to review the different rules that can be used to prove two triangles congruent.

1. **Side-Side-Side (SSS):** If the all corresponding sides of 2 triangles are congruent, then the triangles are congruent.

   ![SSS Diagram](image1)

2. **Side-Angle-Side (SAS):** This rule consists of two parts. First, two corresponding sides of the two triangles must be congruent. Second, the angles made by those two sides must be congruent.

   ![SAS Diagram](image2)

3. **Angle-Side-Angle (ASA):** This rule also consists of two parts. First, two corresponding angles of the two triangles must be congruent. Second, the line segments that connect the two angles must be congruent.

   ![ASA Diagram](image3)

4. **Angle-Angle-Side (AAS):** Two angles of two triangles must be congruent. In addition, one non-included side of the 2 triangles must be congruent.

   ![AAS Diagram](image4)
Exercise #25

Standard: Geometry and Spatial Sense

Indicator: Geometry #6 (Grade 7)
Determine and use scale factors for similar figures to solve problems using proportional reasoning.

Tina has a triangular garden in her front lawn. She wants to make the garden bigger so that the side of the garden that touches the driveway is 9 feet long. However, she wants to keep the shape of her garden exactly the same.

a. What is the ratio of any side of her current garden to the corresponding side of her planned garden?

b. How long should the sidewalk edge of her new garden be so that the shape of the garden remains the same? Explain your answer using words, numbers, or symbols.
Scoring Guide

Point Value: 3

3 Points: A complete response contains the following elements.
- Student indicates that the ratio of the side of the old garden to the side of the new garden is 1:3 or 3:9 (1 to 3, 3 to 9, etc.).
- Student calculates that the sidewalk edge of the new garden should be 12 feet in length.
- Student reasoning shows use of proportions to find the new length of the sidewalk edge of the garden.

2 Points: Response includes 2 of 3 elements of the complete response.

1 Point: Response includes 1 of 3 elements of the complete response.

0 Points: Response includes 0 of 3 elements of the complete response.
Exercise #26

**Standard:** Geometry and Spatial Sense

**Indicator:** Geometry #7 (Grade 7)
Identify the line and rotation symmetries of two-dimensional figures to solve problems.

1. What, if any, angles of rotational symmetry does this figure have?

   ![Sun Image]

   - a. $90^\circ, 180^\circ, 270^\circ$
   - b. $45^\circ, 135^\circ, 225^\circ, 315^\circ$
   - c. $45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ$
   - d. No rotational symmetry

2. Tell how many times this snowflake will display rotational symmetry within one full rotation. List all of the angles of rotational symmetry.

   ![Snowflake Image]
Answer Key
Point Value: 2

1. C

2. See scoring guide below.
   2 Points: A complete response contains the following elements:
   - Response indicates that the snowflake will show rotational symmetry 5 times if it is rotated one full time.
   - Response includes the angles of rotational symmetry: 60°, 120°, 180°, 240°, and 300°
   1 Point: Response contains 1 of 2 elements of the complete response.
   0 Points: Response contains 0 of 2 elements of the complete response.

Teacher Note: Students may find a review of rotational symmetry helpful. A figure has rotational symmetry if it is possible to rotate the figure around its center and obtain a coinciding figure at an angle of less than 360°. For example, if you rotate the 4-point star shown 90°, you obtain a shape that is exactly the same as the starting figure. So, the figure has rotational symmetry. In total, the figure has three angles of rotational symmetry: 90°, 180°, and 270°. We don't count 360° when we look at rotational symmetry because a rotation of 360° returns a figure to its original state.
Exercise #27

Standard: Geometry and Spatial Sense

Indicator: Geometry #7 (Grade 7)
Identify the line and rotation symmetries of two-dimensional figures to solve problems.

1. Complete the figure below so that the dotted line becomes a line of symmetry.

2. Look at the figure below.

How many turns of rotational symmetry does this figure have?

__________________

Identify the number of degrees of rotation (from the initial figure) for each turn.

__________________
Scoring Guide

Point Value: 3 Points (1 + 2)

1. Student completes the figure.

2. 2 Points: Student response contains these 2 elements:
   - Student identifies that there are 5 turns of rotational symmetry;
   - Student gives the degrees of the turns: 60°, 120°, 180°, 240°, and 300°.

   **NOTE:** It is OK if the student identifies 6 turns and includes 360°.

   1 Point: Response contains 1 of 2 elements of the 2-point response.

   0 Points: Response contains 0 of 2 elements of the 2-point response.

**Teacher Note:** A plane figure has *line symmetry* when it is possible to draw a line through the figure so that the two halves of the figure are mirror images of one another. The line that divides the figure into halves is the *line of symmetry*. Any figure that does not have symmetry exhibits *asymmetry*, or is asymmetrical.
Exercise #28

1. Janeesha drove 550 kilometers from Cleveland to Chicago. She left Cleveland at 8:30 a.m. and arrived in Chicago at 2 p.m. Which is the most appropriate way to describe Janeesha’s average speed during the trip?

   O a. kilometers per minute
   O b. kilometers per hour
   O c. minutes per kilometer
   O d. hours per kilometer

2. Jamal used a garden hose to fill his 500 gallon fish pond. At 7:00 a.m. the pond is empty. At 7:50 a.m. the pond is full. Which is the most appropriate way to describe the average water flow rate from the garden hose?

   O a. gallons per hour
   O b. minutes per gallon
   O c. ounces per minute
   O d. gallons per minute

3. In 2000, Rosco’s Burgers started out with 7 restaurants. By 2005, Rosco’s Burgers had expanded to have 16 restaurants. Which is the best way to describe the growth rate of the Rosco’s Burgers chain?

   O a. stores per year
   O b. stores per month
   O c. months per store
   O d. years per store
Answer Key
Point Value: 3 (1 point each)

1. B kilometers per hour
2. D gallons per minute
3. A stores per year

Teacher Note:
Unlike many types of math problems, the problems that measure this indicator do not necessarily have a right or a wrong answer; rather, they tend to have a “best” answer and distracters that may be right but that are inappropriate for the particular application.

For instance, a student might select “ounces per minute [C]” as the answer to question 2. While oz./min. is a correct way to measure flow rate, it is not as appropriate to this particular application as is gallons per minute. A measurement of 50 gallons per minute makes more sense than does a measurement of 6400 fluid ounces per minute, especially when the vessel being filled has a volume of 500 gallons.
Exercise #29

Standard: Measurement

Indicator: Measurement #2 (Grade 7)
Convert units of area and volume within the same measurement system using proportional reasoning and a reference table when appropriate.

A baseball diamond has an area of 8100 ft². What is the area of a baseball diamond when measured in yd²? Show your work using numbers, words, or symbols.
Scoring Guide

Point Value: 3

3 Points: A complete response includes the following elements

- Student work shows the conversion from square feet to square yards using any combination of numbers, words, or symbols. For example, a response might include an explanation that the number of square yards is 1/9 of the number of square feet; OR explanation that ratio of square yards to square feet is 1:9 (or $1^2$ to $3^2$); OR calculations.

\[
\frac{1 \text{ sq. yd.}}{9 \text{ sq. ft.}} = \frac{x \text{ sq. yd.}}{8100 \text{ sq. ft.}}
\]

\[
x = 900 \text{ square yards}
\]

- Student gives the correct answer—the area of the baseball diamond is 900 square yards.

- Student provides the correct label with the numerical answer (square yards).

2 Points: Student work contains 2 of 3 elements of the complete response.

1 Point: Student work contains 1 of 3 elements of the complete response.

0 Points: Student work contains 0 of 3 elements of the complete response.
Exercise #30

Standard: **Measurement**

Indicator: Measurement #4 (Grade 7)
Solve problems involving proportional relationships and scale factors.

1. An architect made a scale model of a new elementary school. The scale of the model is 1 in. = 3 ft. If the gym in the model is 15 inches long by 21\(\frac{1}{3}\) inches wide, what are the actual dimensions of the gym? Make sure to show your work using numbers, words, or symbols.

2. In the same school, the actual ceilings are 9 feet tall. How high are the ceilings in the scale model?
   
   O a. 3 feet  
   O b. 27 inches  
   O c. 3 inches  
   O d. 9 inches
Scoring Guide

Point Value: 5 (4 points + 1 point)

1. 4 Points: Student response includes these elements.
   - Student gives the correct length of 54 feet.
   - Student gives the correct width of 64 feet.
   - Student response shows use of the given scale to find the measurements of the gym by using any combination of numbers, words, or symbols. For example, the student explains that the number of feet will be three times the number of inches shown on the model. Or, student might include calculations.

\[
\begin{align*}
3 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} &= 36 \text{ in} \\
\frac{1 \text{ in}}{36 \text{ in}} &= \frac{15 \text{ in}}{x \text{ in}} \\
x &= 540 \text{ inches} \\
540 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} &= 45 \text{ feet}
\end{align*}
\]

\[
\begin{align*}
\frac{1 \text{ in}}{36 \text{ in}} &= \frac{64 \text{ in}}{3 \times \text{ in}} \\
x &= 764 \text{ inches} \\
764 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} &= 64 \text{ feet}
\end{align*}
\]
   - Numerical answers have the correct labels.

3 Points: Student work contains 3 of 4 elements of the complete response.

2 Point: Student work contains 2 of 4 elements of the complete response.

1 Points: Student work contains 1 of 4 elements of the complete response.

0 Points: Student work contains 0 of 4 elements of the complete response.

2. C 3 inches
Exercise #31

Every day, Tricia jogs along the same path. On average, it takes her 6336 strides to run the entire length of the trail.

a. On an average day, Tricia can run the length of her jogging path in 24 minutes. Find Tricia’s speed in units of strides per second.

b. Tricia wants to compare her speed to Tony’s speed, which is 6.5 miles per hour. Tricia measured that each of her strides is 2.5 feet in length. Find Tricia’s speed in miles per hour. (*Hint: 1 mile = 5280 feet)
Scoring Guide
Point Value: 4

Part A
2 Points: A complete response contains the following elements. Explanations may use any combination of numbers, words, or symbols.
- Response shows the process of converting the given measurement into the rate strides/second. (See example below)
- Student gives the correct response of 4.4 strides per second.

\[
24 \text{ minutes} \times \frac{60 \text{ seconds}}{\text{minute}} = 1440 \text{ seconds} \\
6335 \text{ strides} \div 1440 \text{ seconds} = 4.4 \text{ strides/second}
\]

1 Points: Student work contains 1 of 2 elements of the complete response.
0 Points: Student work contains 0 of 2 elements of the complete response.

Part B
4 Points: A complete response contains the following elements. Explanations may use any combination of numbers, words, or symbols.
- Response shows how to find the rate in miles/hour with the provided measurements. (See example below)
- Student gives the correct response of 7\(\frac{1}{2}\) miles per hour.

\[
6336 \text{ strides} \times \frac{2.5 \text{ feet}}{\text{stride}} = 15840 \text{ feet} \\
15840 \text{ feet} \div 5280 \text{ feet/mile} = 3 \text{ miles} \\
24 \text{ minutes} \div 60 \text{ minutes/hour} = \frac{2}{5} \text{ hours} \\
3 \text{ miles} \div \frac{2}{5} \text{ hours} = 3 \times \frac{5}{2} = 7 \frac{1}{2} \text{ miles per hour}
\]

1 Points: Student work contains 1 of 2 elements of the complete response.
0 Points: Student work contains 0 of 2 elements of the complete response.

Teacher Note: This item illustrates the arbitrary nature of units of measure. Even though nearly anything can be a unit of measure, we need standard ones that allow us to compare various measurements.
Exercise #32

Standard: Data Analysis and Probability

Indicator: DAP #2 (7th Grade)
Analyze how decisions about graphing affect the graphical representation.

The chart below gives the retail price of regular gasoline in the United States (to the nearest cent).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Per Gallon</td>
<td>$0.93</td>
<td>$0.95</td>
<td>$1.16</td>
<td>$1.13</td>
<td>$1.11</td>
<td>$1.23</td>
<td>$1.06</td>
<td>$1.51</td>
</tr>
</tbody>
</table>

Yong made a graph of the data but left off the labels for the y-axis. Label the intervals on the y-axis so that the gas prices are correctly plotted.
Scoring Guide
Point Value: 2 Points

2 Points: The response includes labels on the interval y-axis as given below.

```
Year          Price per Gallon
1986          $0.80
1988          $1.00
1990          $1.20
1992          $1.40
1994          $1.60
1996          $0.80
1998          $1.00
2000          $1.20

```

1 Point: The response includes one of the following features:
- The minimum value on y-axis is $0.00 and all of the interval values on it are evenly spaced apart OR
- The top half of the graph is labeled correctly from $0.80 to $1.60

0 Points: The response includes neither of the features of a 1-point response.

Teacher Note: The purpose of this item is to help students identify graphs with misleading scales and to give them practice creating graphs with appropriate scales. Two frequent mistakes can make a graph misleading. (1) The y-axis does not begin at zero. This can distort the changes shown in the graph. (2) The scale does not increase by a consistent amount. This also distorts the changes shown in the graph. Graphs are helpful because they help us show change. Distorting these changes can misrepresent the facts.

Example of a scale that does not begin at zero. The changes in price look bigger than they really are.

These types of mistakes appear often in the newspaper. Students can find some instances and discuss why the graphs are misleading.
Exercise #33

Standard: Data Analysis and Probability

Indicator: DAP #6 (7th Grade)
Identify misuses of statistical data.

The KleanFresh company asked five dentists which toothpaste they preferred – Kleanfresh or a generic brand. Four of the dentists chose Kleanfresh. As a result, Kleanfresh began advertising in their new TV commercial that “4 out of 5 dentists surveyed prefer Kleanfresh.” Identify 2 reasons why this advertisement is misleading.
Scoring Guide

Point Value: 2 Points

2 points  The response includes both of the following elements:
  - The response indicates that the advertisement makes it seem like there are a large number of dentists who prefer Kleanfresh (or that many dentists were sampled), but they actually only used a sample of 5 dentists.
  - The response indicates that the advertisement makes it seem like these dentists prefer Kleanfresh above all other toothpastes but they only had them choose between 2 toothpastes.

1 point  The response includes one of the elements of a 2-point response.

0 points  The response includes none of the elements of a 2-point response.
Exercise #34

Standard:  Data Analysis and Probability

Indicator:  DAP #2 (7th Grade)
Analyze how decisions about graphing affect the graphical representation.

Percent of Students Receiving A’s

a. The graph above shows the percent of students receiving A’s within each grade level at a middle school. Tell why this graph is misleading.

b. What would you do to the graph to make it a more accurate representation of the data?
Scoring Guide

Point Value: 2 Points

2 points The response includes both of the following elements:

- The response gives a reasonable explanation why the graph is misleading: The response indicates that the graph makes it seem like a much higher percent of 6^{th} graders that got A’s than did 7^{th} graders and 8^{th} graders.;
- The response includes a reasonable solution: The graph should have a more reasonable scale on the y-axis (such as 0 to 50).

1 point The response includes one of the elements of a 2-point response.

0 points The response includes none of the elements of a 2-point response.
Exercise #35

Serita made a graph of historical gas prices for her local newspaper. Her graph is below.

Gas Prices (Price per Gallon)

Give 2 reasons why Serita’s graph might be misleading?
Scoring Guide

Point Value: 2 Points

2 points: The response has both of the following elements:

- The intervals on the y-axis are not equal (i.e. it looks like the differences between the gas prices labeled on the y-axis are equal because they are equally spaced apart);
- The y-axis should begin at zero or show that the graph extends below $0.93.

1 point: The response has one of the elements of a 2-point response.

0 points: The response has none of the elements of a 2-point response.

Teacher Note: Here is an example of a graph that corrects the two misleading elements of the original graph.
Exercise #36

Standard: Geometry and Spatial Sense

Indicator: Geometry #2 (Grade 7)
Determine sufficient (not necessarily minimal) properties that define a specific two-dimensional figure or three-dimensional object.

The three properties below describe a certain type of geometric figure.

1. This figure has exactly three sides.
2. This figure has two congruent angles.
3. The sum of two of the figure’s angles is 90°.

Using the three properties, tell whether each of the figures below matches the properties. Explain your reasoning using the three properties.

a. 

b.


**Scoring Guide**

Point Value: 2

**Part A**

2 Points: Response contains the following information:

- Statement that the triangle does not match the three properties.
- Explanation of why the triangle does not fit the properties.
  Possibilities include: the triangle is not a right triangle; OR since the triangle has an obtuse angle, the measures of the other two angles must add up to less than 90 degrees.

1 Point: Response contains 1 of the 2 parts of the 2-point response.

0 Points: Response contains 0 of the 2 parts of the 2-point response.

**Part B**

2 Points: Response contains the following information:

- Statement that the triangle matches the three properties.
- Explanation of why the triangle fits the properties. Possibilities include: it is a right isosceles triangle; OR it is a triangle with two congruent sides and two angles whose measures add up to 90 degrees.

1 Point: Response contains 1 of the 2 parts of the 2-point response.

0 Points: Response contains 0 of the 2 parts of the 2-point response.
Exercise #37

Standard: Measurement

Indicator: Measurement #6 (Grade 7)
Use strategies to develop formulas for finding area of trapezoids and volume of cylinders and prisms.

1. What is the area of the trapezoid?

   a. 120 cm$^2$
   b. 130 cm$^2$
   c. 240 cm$^2$
   d. 250 cm$^2$

2. Tina made a trapezoid by putting 7 identical isosceles triangles together. The base of each triangle has a length of $x$ and the height of each triangle is $2x$. What is the area of the trapezoid? Show your work.
Scoring Guide

Point Value: 3

1. **d.** 250 cm²

2. See guidelines below.

2 Points: A complete response contains the following elements.

- Student calculates that the area of the trapezoid is 7x² square units.
- The student shows an appropriate method for calculating the area.
  - Example 1: The area of one triangle is x², so the area of all 7 triangles is 7x².
  - Example 2: The student divides the figure up differently and finds the composite area.

1 Point: Contains 1 of 2 elements of the complete response.

0 Points: Contains 0 of 2 elements of the complete response.
Exercise #38

Standard: Patterns, Function and Algebra
Indicator: PFA #8 (7th Grade)
Use formulas in problem solving situations.

The formula for changing Celsius temperature to Fahrenheit is \( F = \frac{9}{5}C + 32 \). Use this formula to find the Fahrenheit temperature equivalent to 45°C. Show or explain your answer using numbers, symbols or words.
Scoring Guide

Point Value: 2 Points

2-point Response indicates:

• The equivalent Fahrenheit temperature is 113°F. (NOTE: It is not necessary to use the °F symbol.)
• Clearly shows or explains how the formula was used to find the Fahrenheit equivalent. For example, but not limited to:
  The student shows:
  \[ F = \frac{9}{5} \times 45 + 32; F = 81 + 32; F = 113 \]

  OR Explains that they multiplied 45 by \( \frac{9}{5} \) and then added 32
  OR Explains that they multiplied 45 by 9, divided by 5, and then added 32

1-point Response contains all elements of a 2 point response with an error in ONE of the bulleted score points.

0-points No response OR response contains TWO or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #39

Standard: Measurement

Indicator: Measurement #7 (Grade 7)
Develop strategies to find the area of composite shapes using the areas of triangles, parallelograms, circles and sectors.

Explain how you would find the area of each composite shape. Then, compute the area using $\pi \approx 3.14$. Dots mark the centers of circles.

1. 

2. Find the area of the shaded portion.

2. 

Scoring Guide

Point Value: 4 (2 points + 2 points)

Question 1:
2 Points: Response contains the following elements.
- Student explains that you need to find the sum of the area of the semicircle and the area of the triangle.
- Student computes the area \((18\pi + 30 \text{ mm}^2 \approx 86.5 \text{ mm}^2)\).
1 Point: Contains 1 of 2 elements of the complete response.
0 Points: Contains 0 of 2 elements of the complete response.

Question 2:
2 Points: Response contains the following elements.
- Student explains that you need to find area of the circle and then subtract it from the area of the square.
- Student computes the area \((64 - 16\pi \text{ in}^2 \approx 13.8 \text{ in}^2)\).
1 Point: Contains 1 of 2 elements of the complete response.
0 Points: Contains 0 of 2 elements of the complete response.
Exercise #40

Standard: Geometry and Spatial Sense

Indicator: Geometry #8 (Grade 7)
Perform translations, reflections, rotations and dilations of two-dimensional figures using a variety of methods.

Look at the figure below. Perform the following geometrical transformations and draw the resulting figure on the grid provided.

1. Rotate the figure 90°.
2. Dilate the figure by a factor of 3.
Scoring Guide

Point Value: 2

2 Points: The complete response contains the following 2 elements (the order does not matter):
- Resulting figure is rotated by 90°;
- Resulting figure is enlarged by a factor of three.

1 Point: Response contains 1 of 2 elements of the complete response.

0 Points: Response contains neither element of the complete response.
Exercise #41

Standard: Geometry and Spatial Sense

Indicator: Geometry #8 (Grade 7)
Perform translations, reflections, rotations and dilations of two-dimensional figures using a variety of methods.

Figure ABCD below has the following coordinates:

A (-10,7); B (-2,4); C (-5,1); D (-9,1).

1. Reflect figure ABCD along the y-axis to obtain figure A₁B₁C₁D₁. Label all four points.
2. Translate figure A₁B₁C₁D₁ by decreasing the y value of each point by 9. Label your new figure as A₂B₂C₂D₂.
Scoring Guide

Point Value: 4

4 Points: A complete response contains the following elements:
- Student reflects $ABCD$ across the y-axis to obtain $A_1B_1C_1D_1$
- Student translates $A_1B_1C_1D_1$ to obtain $A_2B_2C_2D_2$
- Student labels $A_1B_1C_1D_1$ correctly.
- Student labels $A_2B_2C_2D_2$ correctly

3 Points: Response includes 3 of 4 elements of the complete response.

2 Points: Response includes 2 of 4 elements of the complete response.

1 Point: Response includes 1 of 4 elements of the complete response.

0 Points: Response includes 0 of 4 elements of the complete response.

Teacher Note: Subscript notation is used similarly to prime ($A'$, etc).
Exercise #42

A cone with a height of 9 inches and a base with a diameter of 6 inches stands on its base.

1. Make a sketch of the cone and label its dimensions.
2. How will the cone look when it is viewed from the top? Draw and label a sketch of this top-view perspective.
3. How will the cone look when it is viewed from the front. Draw and label a sketch of this front-view perspective.
Scoring Guide

Point Value: 4

4 Points: A complete response contains the following elements:
- Student draws the three-dimensional view properly (see below);
- Student labels the dimensions of the three-dimensional view (see below);
- Student draws a front view of the cone;
- Student draws a top view of the cone. (NOTE: student may or may not include the dot representing the top of the cone.)

3 Points: Response includes 3 of 4 elements of the complete response.

2 Points: Response includes 2 of 4 elements of the complete response.

1 Point: Response includes 1 of 4 elements of the complete response.

0 Points: Response includes 0 of 4 elements of the complete response.
Exercise #43

Standard: Measurement

Indicator: Measurement #3 (Grade 7)
Estimate a measurement to a greater degree of precision than the tool provides.

1. Estimate the length of the arrow to the nearest ¼ inch.

   ![Arrow Diagram]
   
   0 1 2 3 4 5 inches

   O. a. 3 inches
   O. b. 3¼ inches
   O. c. 3½ inches
   O. d. 3¾ inches

2. Which measurement is closest to the exact length of the paper clip?

   ![Paper Clip Diagram]
   
   0 1 2 3 4 5 inches

   O a. 1¾ inches
   O b. 2 inches
   O c. 2¾ inches
   O d. 4½ inches
Answer Key

Point Value: 2 (1 point each)

1. B 3¼ inches
2. A 1¾ inches
Exercise #44

Standard: Measurement

Indicator: Measurement #3 (Grade 7)
Estimate a measurement to a greater degree of precision than the tool provides.

1. An angle is shown on the protractor above. Estimate the measure of the angle to the nearest degree.

   O  a.  77°
   O  b.  68°
   O  c.  63°
   O  d.  67°
2. Rosa and Phillip both used this ruler to measure the length of this pencil.

Rosa estimated the length of the pencil as 7.5 cm. Phillip estimated the length of the pencil as 7.64 cm. Which of the two measurements represents an estimate? Why?
Scoring Guide

Point Value: 3 (1 Point + 2 Points)

1. C 63°

2. 2 Points: A complete response contains these two elements.
   - Explanation that Rosa’s measurement represents an estimate.
     (NOTE: it is acceptable if the student explains that 7.5 cm represents an estimate, but does not mention Rosa’s name.)
   - Justification of why Rosa’s measurement is an estimate. For example, student explains that 7.64 is too precise in this case to be an estimate. It seems like an estimate.

   1 Point: Response contains 1 of 2 elements of the complete response.

   0 Points: Response contains 0 of 2 elements of the complete response.
Exercise #45

Standard: Measurement

Indicator: Measurement #6 (Grade 7)
Use strategies to develop formulas for finding area of trapezoids and volume of cylinders and prisms.

1. What is the formula for finding the volume of this triangular prism?
   - a. \( \frac{1}{2} xbh \)
   - b. \( \frac{1}{2} bh \)
   - c. \( bhx^2 \)
   - d. \( \frac{1}{2} bhx^2 \)

2. What is the volume of the cylinder at the right?
   Give the formula you use to find the volume.

   - Volume of a cylinder: \( \pi r^2 h \)
Scoring Guide

Point Value: 5

1. O a. \( \frac{1}{2} xbh \)

2. See guidelines below.

4 Points: Student answer contains the attributes shown below.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>Student gives the correct formula for the volume of a cylinder: ( V = \pi r^2 h ), where ( r ) is the radius and ( h ) is the height. As long as the student explains what the variables mean, they do not have to be ( r ) and ( h ).</td>
</tr>
<tr>
<td>Step 2</td>
<td>Student substitutes the correct numbers for ( r ) and ( h ), which are 3 mm and 14 mm, respectively.</td>
</tr>
<tr>
<td>Step 3</td>
<td>Student solves for the volume, which is ( 126\pi ) mm(^3) or 395.64 mm(^3) (if ( \pi ) is estimated as 3.14).</td>
</tr>
<tr>
<td>Step 4</td>
<td>Student includes the appropriate label with his or her answer, which is mm(^3) or cubic millimeters.</td>
</tr>
</tbody>
</table>

3 Points: Student response contains 3 of the 4 necessary attributes listed above.

2 Points: Student response contains 2 of the 4 necessary attributes listed above.

1 Point: Student response contains 1 of the 4 necessary attributes listed above.

0 Points: Student response contains none of the necessary attributes listed above.
Exercise #46

Standard: Measurement

Indicator: Measurement #7 (Grade 7)

Develop strategies to find the area of composite shapes using the areas of triangles, parallelograms, circles and sectors.

Explain how you would find the area of each composite shape. Then, compute the area using $\pi \approx 3.14$. Dots mark the center of circles.

1. 

2. Find the area of the shaded portion.

2. Find the area of the shaded portion.
Scoring Guide
Point Value: 4 (2 points + 2 points)

Question 1:
2 Points: Response contains the following elements.
  • Student explains that you need to find the sum of the area of the ¾ of a circle with radius 8 cm, the area of a square with side length 8 cm, and the area of a triangle with base 8 cm and height 8 cm.
  • Student computes the area \((48\pi + 64 + 32 \text{ cm}^2 \approx 246.7 \text{ cm}^2)\).
1 Point: Contains 1 of 2 elements of the complete response.
0 Points: Contains 0 of 2 elements of the complete response.

Question 2:
2 Points: Response contains the following elements.
  • Student explains that you need to find area of the small circle and then subtract it from the area of the large circle.
  • Student computes the area \((100\pi - 25\pi \text{ mm}^2 = 75\pi \text{ mm}^2 \approx 235.5 \text{ mm}^2)\).
1 Point: Contains 1 of 2 elements of the complete response.
0 Points: Contains 0 of 2 elements of the complete response.
Exercise #47

Standard: Measurement

Indicator: Measurement #8 (Grade 7)
Understand the difference between surface area and volume and demonstrate that two objects may have the same surface area, but different volumes or may have the same volume, but different surface areas.

XPress Shipping needs a box in the shape of a rectangular prism and with a volume of exactly 512 cm³. The surface area needs to be as small as possible. One possible box is shown.

Describe or sketch a different box that has the same volume as Box A, and a surface area less than that of Box A. Show work or provide an explanation to verify that the new box meets the criteria.
Scoring Guide

Point Total: 2

2 Points: Student sketches or describes a box that has the same volume as Box A but a smaller surface area. For example, student describes a box with dimensions 8 cm x 8 cm x 8 cm. Or, student draws a box like the one shown.

![Diagram of a box with dimensions 8 cm x 8 cm x 8 cm]

Student explains verifies that his or her new box meets the criteria in the prompt. For example, the cube shown has a volume of 512 cm$^3$. While Box A has a surface area of 448 cm$^2$, this box has a surface area of only 384 cm$^2$. **NOTE: Other dimensions are acceptable as long as they meet the criteria in the prompt.**

1 Point: Student provides a description or sketch of a box with a volume of 512 cm$^3$ and a surface area smaller than 448 cm$^2$. Student does not show work or give an explanation to verify that the box meets the required criteria.

0 Points: No response or incorrect response.
Exercise #48

Standard: Measurement

Indicator: Measurement #8 (Grade 7)
Understand the difference between surface area and volume and demonstrate that two objects may have the same surface area, but different volumes or may have the same volume, but different surface areas.

Russell needs to find a box for his friend’s birthday present, but he only has enough wrapping paper to cover a surface area of 216 square inches. Russell needs a box with a volume as large as possible. One possible box is shown.

```
12 in.
2 in.
16 in.
```

Box A

Sketch or describe a different box that has the same surface area as Box A, but a volume larger than that of Box A. Show work or provide an explanation to verify that the new box meets the criteria.
Scoring Guide

Point Total: 2

2 Points: Student sketches or describes a box that has the same surface area as Box A but a larger volume. For example, student describes a box with dimensions 6 in. x 6 in. x 6 in. Or, student draws a box like the one shown.

![Diagram of a box with dimensions 6 in. x 6 in. x 6 in.]

Student explains verifies that his or her new box meets the criteria in the prompt. For example, the cube shown has a surface area of 216 square inches. While Box A has a volume of 144 in³, this box has a volume of 216 in³. **NOTE: Other dimensions are acceptable as long as they meet the criteria in the prompt.**

1 Point: Student provides a description or sketch of a box with a surface area of 216 in² and volume larger than 144 in³. Student does not show work or give an explanation to verify that the box meets the required criteria.

0 Points: No response or incorrect response.
Exercise #49

Standard: Data Analysis and Probability

Indicator: DAP #3 (7th Grade)
Analyze a set of data by using and comparing combinations of measures of center (mean, mode, median) and measures of spread (range, quartile, interquartile range)

Ann’s teacher says that he will give students in his class the choice of getting a grade based on their median quiz scores or on their mean quiz scores. Ann’s quiz scores were 59, 92, 88, 67, 87, 81 and 69.

1. Compare the mean and the median. Explain which grading method Ann should choose.

2. Ann’s teacher decided to throw out her lowest score. Again, compare the median and the mode. Explain which grading method Ann should choose.
Scoring Guide

Point Value: 4 Points (2 points each)

Question 1:

2 Points: The response correctly shows that Ann would want to use her median score.
- The response includes correct calculation of the mean, 78 (or 77.6);
- The response includes correct identification of the median, 81.

1 Point: The response includes an incorrect mean OR the response includes an incorrect median.

0 Point: The response includes an incorrect mean and median.

Question 2:

2 Points: The response correctly shows that Ann would still want to use her median score.
- The response includes correct calculation of the mean, 81 (or 80.7);
- The response includes correct identification of the median, 84 (the average of 81 and 87).

1 Point: The response includes an incorrect mean OR the response includes an incorrect median.

0 Point: The response includes an incorrect mean and median.
Exercise #50

Standard: Data Analysis and Probability
Indicator: DAP #3 (7th Grade)
Analyze a set of data by using and comparing combinations of measures of center (mean, mode, median) and measures of spread (range, quartile, interquartile range).

Michael was in charge of collecting contributions for a local charity. He received donations of $25, $100, $40, $25, and $90. Another potential donor wanted to give an amount that was in line with the other donations, so he asked Michael what is an acceptable amount to give.

a. Find the mean, median, mode and range of the donations.
b. How should Michael respond to the question, “what is an acceptable amount to give?” Use your results above to justify your answer.
Scoring Guide

Point Value: 6 Points

6 Points: The response contains the following elements:

- The response correctly identifies the mean = $56;
- The response correctly identifies the mode = $25;
- The response correctly identifies the median = $40;
- The response correctly identifies the range ($25 - $100);
- The response indicates one of the values above as an acceptable amount to give;
- The response justifies the value given.

Examples:

1. Michael may want to tell the donor that $56 is an acceptable amount if he wants to get the largest donation.
2. Michael should tell the donor $25 since that is what the most number of people gave.

5 points: The response contains 5 of the elements of a 6-point response.

4 points: The response contains 4 of the elements of a 6-point response.

3 points: The response contains 3 of the elements of a 6-point response.

2 points: The response contains 2 of the elements of a 6-point response.

1 point: The response contains 1 of the elements of a 6-point response.

0 points: The response contains none of the elements of a 6-point response.
Shauna babysits the neighbor’s girls after school 2 days a week and makes $20 each week for the first three weeks of February. Her neighbor has to work overtime everyday during the last week of the month, so Shauna babysits the girls all 5 days and makes $60.

Compare Shauna’s average weekly earnings for the first 3 weeks with her average weekly earnings for all 4 weeks. How does adding the last week change her average earnings?
Scoring Guide

Point Value: 4 Points

4 Points: Response contains the following:
  o Average for the 1st three weeks of February ($20);
  o Average for all four weeks ($30);
  o Explanation that adding the fourth week increases the average weekly earnings by $10. (If student did not calculate the correct mean and median, this part might be different.);
  o Student work to shows the correct process for answering the prompt.

3 Points: Response contains an error in 1 part of the full response.

2 Points: Response contains errors in 2 parts of the full response.

1 Point: Response contains errors in 3 parts of the full response.

0 Points: Incorrect or no response.
Exercise #52

Standard: Data Analysis and Probability

Indicator: DAP #4 (7th Grade)
Construct opposing arguments based on analysis of the same data, using different graphical representations.

The school newspaper staff conducted monthly polls of which mayoral candidate students supported in the May election. The newspaper staff wanted to put a graph of the survey results in the newspaper before the election in its May issue.

<table>
<thead>
<tr>
<th>Candidate</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gannon</td>
<td>59%</td>
<td>57%</td>
<td>57%</td>
<td>44%</td>
<td>31%</td>
<td>50%</td>
</tr>
<tr>
<td>Prixley</td>
<td>41%</td>
<td>43%</td>
<td>43%</td>
<td>56%</td>
<td>69%</td>
<td>50%</td>
</tr>
</tbody>
</table>

Jeremy made the following graph:

Kwame made the following graph:
a. If the newspaper printed Jeremy’s graph, what conclusion might readers make about how students have felt about the two candidates?

b. If the newspaper printed Kwame’s graph, what conclusion might readers make about how students have felt about the two candidates?

c. Which graph do you think the newspaper should run and why?
Scoring Guide
Point Value: 4 Points

4 points  The response has all 4 of the following elements:
- The response indicates that with Jeremy’s graph the readers would conclude that the candidates had equal student support;
- The response indicates that with Kwame’s graph the readers would conclude that the students’ support of the 2 candidates changed over time and currently Prixley had the most support;
  - Example: At the beginning Gannon had more student support than Prixley but he lost student support and starting in April, Prixley was ahead of Gannon in the polls.
- The response indicates that the newspaper should run Kwame’s graph;
- The response includes a reasonable explanation of why the newspaper should use Kwame’s graph.
  - Examples: Kwame’s graph gives more details about what actually happened throughout the whole 5 months.
  - If the newspaper used Jeremy’s graph the readers would be misled to think that the candidates were currently tied in the polls.

Note: A student may get credit for selecting Jeremy’s as the best graph to run in the newspaper if a reasonable explanation is given (e.g. they should use a graph that doesn’t sway students’ opinions.)

3 points  The response has 3 of the elements of a 4-point response.

2 points  The response has 2 of the elements of a 4-point response.

1 point  The response has 1 of the elements of a 4-point response.

0 points  The response has none of the elements of a 4-point response.
Exercise #53

Standard: Patterns, Function and Algebra
Indicator: PFA #7 (7th Grade)
Justify that two forms of an algebraic expression are equivalent, and recognize when an expression is simplified; e.g., \(4m = m + m + m + m\) or \(a \cdot 5 + 4 = 5a + 4\).

Mike says that \(5a - 3(4a + 7)\) is the same as \(-7a - 21\). Is he correct? Justify your answer using pictures, words, numbers or symbols.
Scoring Guide

Point Value: 3 Points

3-points Response indicates:

- Mike is correct.
- An explanation that shows the expression first simplified using the distributive property. For example, $5a – 3(4a + 7)$ is the same as $5a – 12a – 21$.
- An explanation that shows the expression then simplified by combining like terms. For example, $5a – 12a – 21$ is the same as $(5a – 12a) – 21$ which is the same as $–7a – 21$.

2-points Response contains all elements of a 3 point response with an error in ONE of the bulleted score points.

1-points Response contains all elements of a 3 point response with an error in TWO of the bulleted score points.

0-points No response OR response contains THREE or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #54

Standard: Patterns, Function and Algebra

Indicator: PFA #7 (7th Grade)
Justify that two forms of an algebraic expression are equivalent, and recognize when an expression is simplified; e.g., \(4m = m + m + m + m\) or \(a \cdot 5 + 4 = 5a + 4\).

1. Which of the following expressions is equivalent to \((n + n + n) ÷ 5\)?

   O a. \(5n^3\)
   O b. \(5(3n)\)
   O c. \(\frac{n^3}{5}\)
   O d. \(\frac{3n}{5}\)

2. Michael says that \(m^4 = m \cdot m \cdot m \cdot m\) is a true equation. Is he correct? Why or why not?
Scoring Guide
Point Value: 3 Points (1 point + 2 points)

1. O   d. $\frac{3n}{5}$

2. 2 points Response indicates:
   - Michael is correct.
   - An explanation that indicates $m^4$ means that the base $m$ is multiplied by itself 4 times. This is the same as $m \cdot m \cdot m \cdot m$, so they are equivalent equations.

   1 points Response contains 1 of 2 elements of the 2-point response.

   0 points No response OR response contains 0 of 2 elements of the 2-point response.
Exercise #55

Standard: Patterns, Function and Algebra

Indicator: PFA #5 (7th Grade)
Represent linear equations by plotting points in the coordinate plane.

The Hoop It Up gym is running a special. It costs $8.00 to reserve the gym and then $2.50 for each hour. Graph the linear equation on the coordinate plane and plot the points that indicate the cost for renting the gym for 2 hours and 4 hours. Write an equation to show how much it would cost for \(x\) amount of hours. If you and your friends decided to rent the gym for 3 hours, how much would it cost?
Scoring Guide

Point Value: 4 Points

4-points Response indicates:

- A correctly constructed graph with the ordered pairs (2, 13) and (4, 18) plotted on a coordinate plane. For example,

- The line for the equation graphed that passes through the ordered pairs (2, 13) and (4, 18).
- The cost for 3 hours to rent the gym is $15.50.
- An equation to represent the problem situation. For example, but not limited to, an equation that is equivalent to $y = 2.5x + 8$.

3-points Response contains all elements of a 4 point response with an error in ONE of the bulleted score points.

2-points Response contains all elements of a 4 point response with an error in TWO of the bulleted score points.

1-point Response contains all elements of a 4 point response with an error in THREE of the bulleted score points.

0-points No response OR response contains FOUR or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #56

Standard: Patterns, Function and Algebra

Indicator: PFA #5 (7th Grade)
Represent linear equations by plotting points in the coordinate plane.

Andrew’s homework assignment was to construct a graph of the equation
\( y = x - 5 \). He constructed the following graph.

Does this graph represent the equation \( y = x - 5 \)?
Justify your answer.
Scoring Guide

Point Value: 2 Points

2-points Response indicates:

- The graph does NOT represent the linear equation $y = x - 5$.
- An argument or reason is given that the linear equation graphed does not represent $y = x - 5$. For example, but not limited to, the graph would pass through (0, -5) and (5, 0) if it represented $y = x - 5$; OR that the equation graphed is actually $y = x + 3$; etc.

1-points Response contains all elements of a 2 point response with an error in ONE of the bulleted score points.

0-points No response OR response contains FOUR or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #57

Standard: Patterns, Function and Algebra

Indicator: PFA #6 (7th Grade)
Represent inequalities on a number line or a coordinate plane.

The number of winning votes for 8th grade class president is greater than or equal to 550 and less than or equal to 950. Write these inequalities below. Then graph them on the number line.
Scoring Guide

Point Value: 2 Points

2 points  Response indicates:

- Appropriate inequalities for the problem. For example, but not limited to, equivalent inequalities, such as:
  \[ b \geq 550 \text{ and } b \leq 950; \text{ OR } 550 \leq b \leq 950. \]

- The inequality correctly graphed on the number line as shown:

1 points  Response contains all elements of a 2 point response with an error in ONE of the bulleted score points.

0 points  No response OR response contains TWO or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #58

Standard: Patterns, Function and Algebra
Indicator: PFA #6 (7th Grade) Represent inequalities on a number line or a coordinate plane.

Describe the errors in the following graph for the inequality \( y \leq 4x - 3 \). Then correctly redraw the graph in the space below.
Scoring Guide
Point Value: 3 Points

3-points Response indicates:

- Correctly identifies that the line should be solid;
- Correctly identifies that the shading should be to the right or under of the line;
- Correctly draws a graph for the inequality $y \leq 4x - 3$ with the corrections indicated. For example:

![Graph of inequality](image)

2-points Response contains all elements of a 3 point response with an error in ONE of the bulleted score points.

1-points Response contains all elements of a 3 point response with an error in TWO of the bulleted score points.

0-points No response OR response contains THREE or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #59

Standard: Data Analysis and Probability

Indicator: DAP #1 (7th Grade)
Read, create and interpret box-and-whisker plots, stem-and-leaf plots, and other types of graphs.

1. Which of the following statements can be made about the price of gas?
   - A. Gas in Cleveland is usually more expensive than the U.S. average.
   - B. The lowest price of gas in Cleveland was the same as the lowest U.S. average gas price.
   - C. The range of U.S. average gas prices during this 12-month period was between $2.10 and $3.00.
   - D. Gas prices were lower in June 2005 than in May 2006.

2. Between which dates did the largest difference between the price of gas in Cleveland and national average price of gas occur?
   - A. 7/4/05 and 7/18/05
   - B. 8/29/05 and 9/12/05
   - C. 10/24/05 and 11/7/05
   - D. 1/2/06 and 1/16/06
Answer Key

Point Value: 2 Points (1 point each)

1. D
2. C
Exercise #60

Standard: Data Analysis and Probability
Indicator: DAP #1 (7th Grade)

Read, create and interpret box-and-whisker plots, stem-and-leaf plots, and other types of graphs.

Ages of all Relatives at the Johnson Family Reunion

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1. Which of the following statements can be made about the relatives who attended the Johnson family reunion?
   - O A. There was no one below the age of 10.
   - O B. More relatives were in their 20's than in their 60's.
   - O C. More than half the relatives were older than 40 years old.
   - O D. There were 29 relatives in attendance.

2. What is the mode of the relatives' ages at the reunion?
   - O A. 36
   - O B. 39
   - O C. 41
   - O D. 65

3. What is the median age of the relatives at the reunion?
   - O A. 36
   - O B. 39
   - O C. 41
   - O D. 65
Answer Key

Point Value: 3 Points (1 point each)

1.  D
2.  C
3.  B
Exercise #61

Standard: Data Analysis and Probability

Indicator: DAP #1 (7th Grade)
Read, create and interpret box-and-whisker plots, stem-and-leaf plots, and other types of graphs.

This box-and-whisker plot shows the distribution of points scored by Lebron James for the Cleveland Cavaliers in each of the 2005-06 regular season games.

1. What is the median number of points per game scored by Lebron James?
   O  A.  26
   O  B.  30
   O  C.  32
   O  D.  36

2. What is the range of points per game scored by Lebron James?
   O  A.  10
   O  B.  14
   O  C.  38
   O  D.  52

3. What does the interquartile range tell us in this plot?
   O  A.  that Lebron scored between 14 and 32 points in 50% of his games
   O  B.  that Lebron scored between 26 and 36 points in 50% of his games
   O  C.  that Lebron scored between 32 and 36 points in 25% of his games
   O  D.  that Lebron scored between 36 and 52 points in 25% of his games
Answer Key

Point Value: 3 Points (1 point each)

1. C
2. C
3. B

Teacher Notes:

- lower quartile
- upper quartile
- interquartile range
- upper extreme
Exercise #62

Standard: Data Analysis and Probability

Indicator: DAP #5 (7th Grade)
Compare data from two or more samples to determine how sample selection can influence results.

Carlotta’s Chicken is a chain of many fast food restaurants. The company made a survey to find out how much people enjoy the Crispy Chicken Combo. The company surveyed 2 different groups and collected 1,000 responses from each group. One group consisted of people waiting in line at the restaurants. The other group consisted of people contacted at random by telephone.

Based on what you know about the surveys, which graph gives a more accurate representation of how much consumers like the Crispy Chicken Combo? Explain your thinking.
Scoring Guide

Point Value: 2 Points

2 Points: Student response contains both of the following elements:

- The response indicates that the graph of the telephone survey results is probably the more accurate of the two;
- The response includes a reasonable explanation of why the telephone survey results are more accurate.
  - Example: People probably only go to Carlotta’s if they like the food. The survey of people in the store probably only includes people who like the food there.
  - Example: The telephone survey is better because it uses a random group of people. These people are more likely to represent the entire population.

1 Point: The response has 1 of the elements of a 2-point response.

0 Points: The response has none of the elements of a 2-point response.
Exercise #63

Standard: Geometry and Spatial Sense

Indicator: Geometry #3 (Grade 7)
Use and demonstrate understanding of the properties of triangles.

Use the Pythagorean Theorem to find the lengths of the missing sides of the following right triangles.

1.

\[ \text{9} \]
\[ \text{15} \]
\[ \text{x} \]

2.

\[ \text{5} \]
\[ \text{12} \]
\[ \text{y} \]
Scoring Guide

Point Value: 4 (2 x 2 points)

Question 1

2 Points: Student indicates that x = 12 and shows correct work. For example:

\[ x^2 + 9^2 = 15^2 \]
\[ x^2 = 15^2 - 9^2 = 225 - 81 = 144 \]
\[ x = \sqrt{144} \]
\[ x = 12 \]

1 Point: Student indicates that x=12, but does not show complete work. OR
Student makes minor arithmetic error, but follows the correct steps using the Pythagorean Theorem.

0 Points: Student does not provide the correct answer and does not show work that indicates an understanding of the Pythagorean Theorem.

Question 2

2 Points: Student indicates that x = 13 and shows correct work. For example:

\[ 5^2 + 12^2 = y^2 \]
\[ y^2 = 25 + 144 = 169 \]
\[ x = \sqrt{169} \]
\[ x = 13 \]

1 Point: Student indicates that x=13, but does not show complete work. OR
Student makes minor arithmetic error, but follows the correct steps using the Pythagorean Theorem.

0 Points: Student does not provide the correct answer and does not show work that indicates an understanding of the Pythagorean Theorem.
Teacher Notes: Triangles—specifically right triangles—make up one of the most important sets of figures in geometry. Right triangles are particularly important because students can use the Pythagorean Theorem \( (a^2 + b^2 = c^2) \) to solve for missing side lengths.

Students may have trouble understanding why right triangles are special and why the Pythagorean Theorem can be applied only to right triangles. It may be helpful for students to see the Pythagorean Theorem visually.

The diagram above shows why the Pythagorean Theorem works only for right triangles. Right triangles can be made from one side of each of three squares, in this case squares with side lengths 3, 4, and 5. In the Pythagorean Theorem, we take the square of each side length—we basically get the area of each square. By adding the areas of A and B, we get \( 3^2 + 4^2 = 25 \). The sum of the areas of A and B is equal to the area of C: \( 3^2 + 4^2 = 5^2 \). Students might test the theorem out by cutting out squares of paper and making triangles out of them.

The triangles used in this particular Bell Work exercise are common triangles known as Pythagorean Triples. The three side lengths are whole numbers. Common triples include (3,4,5); (5,12,13); (8,15,17); (7,24,25). Once students have a conceptual understanding of right triangles and the Pythagorean Theorem and have gotten used to performing the calculations, students can save time and effort by memorizing these common right triangles.
Exercise #64

Standard: **Geometry and Spatial Sense**

Indicator: Geometry #3 (Grade 7)
**Use and demonstrate understanding of the properties of triangles.**

1. What is the measure of angle $p$?

   ![Diagram of a triangle with angles 114° and 41°]

   - a. 25°
   - b. 66°
   - c. 73°
   - d. 139°

2. Kyra is supposed to draw 4 triangles using the side lengths given in the table below. She was able to draw three triangles, but found that one of the triangles was impossible to draw and could not really be a triangle. Which triangle couldn't she draw?

<table>
<thead>
<tr>
<th>Side A</th>
<th>Side B</th>
<th>Side C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle 1</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Triangle 2</td>
<td>21</td>
<td>8</td>
</tr>
<tr>
<td>Triangle 3</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>Triangle 4</td>
<td>15</td>
<td>23</td>
</tr>
</tbody>
</table>

   - a. Triangle 1
   - b. Triangle 2
   - c. Triangle 3
   - d. Triangle 4
Answer Key
Point Value: 2 (1 point each)

1. A. 25°

2. B. Triangle 2

**Teacher Note:** In item 2, students are asked to figure out the side lengths from which triangles can be created. The rule for checking whether a triangle with given side lengths is possible is simple: the sum of any 2 sides of the triangle must be greater than the 3rd side. For Triangle 2, here is how a student might determine if the triangle is possible.

\[
21 + 8 > 11 \quad \text{TRUE.}
\]
\[
21 + 11 > 8 \quad \text{TRUE.}
\]
\[
11 + 8 > 21 \quad \text{FALSE: Triangle is not possible.}
\]

It might be helpful to remind students that they must check all 3 possibilities for adding the side lengths together.
Eighth Grade Mathematics Bell Work  Teacher Edition

Exercise #65

Standard:  Geometry and Spatial Sense

Indicator:  Geometry #5 (Grade 7)
Apply properties of congruent or similar triangles to solve problems involving missing lengths and angle measures.

QRS and XYZ are similar triangles. The ratio of $\overline{QS}$ to $\overline{XZ}$ is 1 to 4.

What is the measure of $\overline{SR}$? Explain how you know using words, numbers of symbols.
Scoring Guide

Point Value: 2

2 Points: Response includes the following:

- Statement that the measure of SR is four.
- Explanation using any combination of words, numbers, or symbols, for example: SR is \( \frac{1}{4} \) the length of ZY; OR ZY is four times longer than SR; OR shows written calculations.

\[
\frac{1}{4} = \frac{x}{16} \\
4x = 16 \\
x = 4
\]

1 Point: Student response includes 1 of 2 parts of the complete response.

0 Points: Response includes 0 of 2 parts of the complete 2-point response.
Exercise #66

Standard: Geometry and Spatial Sense

Indicator: Geometry #5 (Grade 7)
Apply properties of congruent or similar triangles to solve problems involving missing lengths and angle measures.

1. What is the measure of segment $PQ$?

   a. 1.75
   b. 5.25
   c. 7.00
   d. 15.75

2. What is the measure of segment $QR$?

   a. 2.5
   b. 5.0
   c. 10.0
   d. 22.5
Answer Key

Point Value: 2 (1 point each)

1. C. 7.00

2. A. 2.5

Teacher Note: Students may confuse the different proportional relationships that can be used to find similar attributes of triangles. For instance, students can use part-to-whole ratios, part-to-part ratios, and whole-to-whole ratios. Depending on the lengths that the students are asked to find, they might find some ratios easier to work with than others.

For instance, in Item 1, students might use a whole-to-whole ratio to find the length of segment PQ from segment RT. In this case, the ratio of the small triangle to the large triangle is 9 to 12 (or 3 to 4). In Item 2, the student might use a whole-to-part ratio. In this case, the ratio a student would use is 9 to 3 (or 3 to 1). In order to avoid confusion, students need to understand what the different ratios mean.
Exercise #67

Standard: Number, Number Sense and Operations

Indicator: NNSO #4 (7th Grade)
Use order of operations and properties to simplify numerical expressions involving integers, fractions, and decimals.

1. James is simplifying the expression: \[ \frac{1}{2} - 36 \div 4 \times 2 + 8.5 \]

Which computation should James do first?

O A. subtract 36 from 1/2
O B. divide 36 by 4
O C. multiply 4 times 2
O D. add 2 to 8.5

Use order of operations to find the value of the expressions. Show your work.

2. \[ 6.5 + 16 \div (8 \cdot 4) - 2.5 \]

3. \[ \frac{9}{4} - 0.25 \cdot 8 + 3^2 \]
Scoring Guide

Point Value: 5 Points (1, 2 points, 2 points)

1. B

2. Use 2-Point Scoring Guide

   2 Points: Response includes correct use of order of operations. The student correctly simplifies the expression to 4.5 by including work similar to that shown below.

   \[
   6.5 + 16 \div (8 \cdot 4) - 2.5 = 6.5 + 16 \div 32 - 2.5 \\
   6.5 + 16 \div 32 - 2.5 = 6.5 + 0.5 - 2.5 \\
   6.5 + 0.5 - 2.5 = 4.5
   \]

   1 Point: Response follows the rules for the correct use of order of operations. The student makes a calculation error, but shows the steps used to simplify the expression.

   0 Points: Student response does not reflect an understanding of applying order of operations to algebraic expressions. OR Student work contains two or more calculation errors. OR No response.

3. Use 2-Point Scoring Guide

   2 Points: Response includes correct use of order of operations. The student correctly simplifies the expression to 9.25 by including work similar to that shown below.

   \[
   \frac{9}{4} - 0.25 \cdot 8 + 3^2 = \frac{9}{4} - 2 + 9 \\
   \frac{9}{4} - 2 + 9 = 9 \frac{1}{4}
   \]

   1 Point: Response follows the rules for the correct use of order of operations. The student makes a calculation error, but shows the steps used to simplify the expression.

   0 Points: Student response does not reflect an understanding of applying order of operations to algebraic expressions. OR Student work contains two or more calculation errors. OR No response.
Teacher’s Note:

When reviewing the order of operations, many students benefit from using some type of memory aid for remembering the order of operations. A common one for “PEMDAS” is Please Excuse My Dear Aunt Sally which is used to remember that Parentheses are simplified first, then all Exponents, Multiplication and Division are done next in left to right order, and Addition and Subtraction are computed last in left to right order. Students should be reminded that multiplication and division are done in the order they appear from left to right rather than all multiplication and then all division. The same is true for addition and subtraction. The parentheses step includes all grouping symbols (symbols of inclusion), such as brackets and braces.
Tell whether each of the following is an example of the Associative, Commutative, or Distributive property.

a. \(3(b-5) = (3 \cdot b) - (3 \cdot 5)\) __________________

b. \(4 \cdot 7 = 7 \cdot 4\) __________________

c. \(6(2 + a) = (6 \cdot 2) + (6 \cdot a)\) __________________

d. \((3 \cdot 4) \cdot 7 = 3 \cdot (4 \cdot 7)\) __________________

e. \(80 + y = y + 80\) __________________

f. \((2 + 9)a = 2a + 9a\) __________________

g. \(\frac{3}{8} + \left(\frac{5}{8} + \frac{1}{4}\right) = \left(\frac{3}{8} + \frac{5}{8}\right) + \frac{1}{4}\) __________________

h. \(a + (b+c) = a + (c + b)\) __________________
Answer Key
Point Value: 7 Points (1 point each)

| a. distributive | e. commutative |
| b. commutative  | f. distributive |
| c. distributive | g. associative  |
| d. associative  | h. commutative  |

**Teacher Notes:** There are several properties with which students need to be very comfortable. Students should understand the rules of properties as well as be able to apply those properties in problem situations. A brief description of these follows.

**Commutative Property:** When adding or multiplying numbers, the order in which those numbers are added or multiplied does not affect the sum or product. The word commutative contains the word "commute", which means to go back and forth from one place to another.
- Commutative Property of Addition: $3 + 8 = 8 + 3$, $a + b = b + a$
- Commutative Property of Multiplication: $4 \times 7 = 7 \times 4$, $xy = yx$

**Associative Property:** When adding or multiplying a series of numbers, how those numbers are grouped does not affect the sum or product. The word associative contains the word "associate", which means to be with.
- Associative Property of Addition: $(4 + 5) + 9 = 4 + (5 + 9)$, $(a + b) + c = a + (b + c)$
- Associative Property of Multiplication: $3 \times (7 \times 2) = (3 \times 7) \times 2$, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

**Distributive Property:** When an amount is being multiplied by a quantity in parentheses, every term inside the parentheses is multiplied by that amount. This can be done by combining everything inside the parentheses first and then multiplying or by multiplying everything individually then combining. The word distributive contains the word “distribute”, which means to give something to everyone.
- Distributive Property of Multiplication over Addition: $6(3 + 2 + a) = (6 \times 3) + (6 \times 2) + (6a)$, $6(5 + a) = 30 + 6a$, $a(b + c) = ab + ac$
- Distributive Property of Multiplication over Subtraction: $3(9 – 5) = (3 \times 9) – (3 \times 5)$ or $3(9 – 5) = 3(4)$, $a(b – c) = ab – ac$

**Identity Property:** When the operation is performed on a term, the result is the term. The word identity" means who you are.
- Identity Property of Addition: $8 + 0 = 8$, $a + 0 = a$
- Identity Property of Multiplication: $-12 \times 1 = -12$, $a \cdot 1 = a$

**Inverse Property:** A number combined with another number in any order which results in the identity element of zero or one. Additive inverses are called opposites (the sum is always zero). Multiplicative inverses are called reciprocals (the product is always one).
- Inverse Property of Addition: $10 + -10 = 0$, $a + -a = 0$
- Inverse Property of Multiplication: $2 \cdot \frac{1}{2} = 1$, $a \cdot \frac{1}{a} = 1$

**Zero Property of Multiplication:** Anything multiplied by zero is still zero; $c \cdot 0 = 0$ and $0 \cdot c = 0$. This means that if $xy = 0$, then either $x$ or $y$ or both must be 0.
Exercise #69

Standard: Number, Number Sense and Operations
Indicator: NNSO #5 (7th Grade)

Explain the meaning and effect of adding, subtracting, multiplying and dividing integers.

1. Represent the equation on the number line. \(-7 + -5 = -12\)

2. Represent the equation on the number line. \(-4 + -2 = -6\)

3. N and P are negative integers. Which number line shows the region where the sum of N and P will be located.

   - a.
   - b.
   - c.
   - d.
Scoring Guide
Point Value: 5 Points (2 + 2 + 1)

1. See 2-point scoring guide.
   2 Points: Student draws an arrow from 0 to -7 and a second arrow from -7 to -12. An example is shown.

   ![Arrow from 0 to -7 and a second arrow from -7 to -12]

   1 Point: Student makes a minor mistake in the drawing, but has two arrows pointed in the negative direction. The second arrow begins at the head of the first arrow.

   0 Points: No answer or answer that does not demonstrate an understanding of the concept.

2. See 2-point scoring guide.
   2 Points: Student draws an arrow from 0 to -4 and a second arrow from -4 to -6. An example is shown.

   ![Arrow from 0 to -4 and a second arrow from -4 to -6]

   1 Point: Student makes a minor mistake in the drawing, but has two arrows pointed in the negative direction. The second arrow begins at the head of the first arrow.

   0 Points: No answer or answer that does not demonstrate an understanding of the concept.

3. B

Teacher's Note: The point of this problem is to help students see that the magnitude of the sum of two negative integers is greater than either of the two numbers (the sum is more negative than both of the numbers).
Exercise #70

Standard: Number, Number Sense and Operations

Indicator: NNSO #5 (7th Grade)
Explain the meaning and effect of adding, subtracting, multiplying and dividing integers.

1. Represent the equation on the number line.  
   
   \[-3 + 14 = 11\]

   ![Number line for -3 + 14 = 11]

2. Represent the equation on the number line.  
   
   \[7 + -12 = -5\]

   ![Number line for 7 + -12 = -5]

3. X is a negative integer and Y is a positive integer. Which number line shows the region where the sum of X and Y will be located.

   ![Number lines for options a, b, c, d]
Scoring Guide
Point Value: 5 Points (2 + 2 + 1)

1. See 2-point scoring guide.

   2 Points: Student draws an arrow from 0 to -8 and a second arrow from -8 to 14. An example is shown.

   1 Point: Student makes a minor mistake in the drawing, but has one arrow (beginning at 0) pointed in the negative direction. The second arrow begins at the head of the first arrow and points in the positive direction.

   0 Points: No answer or answer that does not demonstrate an understanding of the concept.

2. See 2-point scoring guide.

   2 Points: Student draws an arrow from 0 to 7 and a second arrow from 7 to -5. An example is shown.

   1 Point: Student makes a minor mistake in the drawing, but has one arrow (beginning at 0) pointed in the positive direction. The second arrow begins at the head of the first arrow and points in the negative direction.

   0 Points: No answer or answer that does not demonstrate an understanding of the concept.

3. C

   Teacher’s Note: The point of this problem is to help students visualize that unlike adding integers of the same sign where the magnitude of the sum increases over the value of the individual integers, when adding a positive and a negative integer, the magnitude of the sum decreases relative to the value of the individual numbers (the sum is closer to zero on the number line).
Exercise #71

Standard: Number, Number Sense and Operations

Indicator: NNSO #5 (7th Grade)

Explain the meaning and effect of adding, subtracting, multiplying and dividing integers.

Number lines give us new ways to describe what happens when we solve multiplication problems.

- Keisha used a number line to model a multiplication problem. \(+2 \times -5\)
  
a. She knew that a negative sign always means “do the opposite of.” So, she knew that she needed to add two sets of the opposite of +5. She showed adding two sets of +5 on a number line.

```
-15 -10 -5 0 5 10 15
```

b. Then, she found the opposite of the first picture. She ended up with \(-10\).

```
-15 -10 -5 0 5 10 15
```

- Carla used a number line to model a different problem. \(-3 \times -4\)
  
a. Carla knew that she had to find the opposite twice. First she showed adding 3 sets of +4 on a number line.

```
-15 -10 -5 0 5 10 15
```

b. Then she showed the opposite of the first number line.

```
-15 -10 -5 0 5 10 15
```

c. Last, she showed the opposite of the opposite, and got the answer +12.
Try using number lines to describe how you solve multiplication problems. Show pictures, explain each step, and give your solution.

1. $-2 \times 3 =$

2. $7 \times 2 =$
Scoring Guide

Total Points: 4 (2 points + 2 points)

Question 1

2 Points: Response includes the following process.

<table>
<thead>
<tr>
<th>Picture Format</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>Step 2</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
</tbody>
</table>

Solution is -6

1 Point: Response contains ONE error in the steps of the complete response.
0 Points: No response OR response contains TWO or more errors in the steps of the complete response OR response demonstrates no understanding of the concept.

Question 2

2 Points: Response includes the following process.

<table>
<thead>
<tr>
<th>Picture Format</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>Step 2</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
</tbody>
</table>

Solution is +14

1 Point: Response contains ONE error in the steps of the complete response.
0 Points: No response OR response contains TWO or more errors in the steps of the complete response OR response demonstrates no understanding of the concept.
Exercise #72

Standard: Number, Number Sense and Operations

Indicator: NNSO #6 (7th Grade)
Simplify numerical expressions involving integers and use integers to solve real life problems.

Simplify the expression.

1. \(2q + 5r\), when \(q = 5\) and \(r = -3\)

2. \(t(2 - 3x) \div 2x\), when \(t = -3\) and \(x = 6\)

3. Joe and his friends are making cupcakes. They baked 62 cupcakes and want to decorate each cupcake with 5 flowers. There are 80 flowers in a box. How many boxes of flowers do they need to buy in order to decorate all of the cupcakes? Show each step of your work.
Scoring Guide

Point Value: 6 (2 points apiece)

Question 1:
2 Points: Student shows written work and simplifies the expression.
\[2q + 5r \rightarrow 2(5) + 5(-3) = 10 - 15 = -5\]
1 Point: Student substitutes numbers correctly but makes a small error in calculation. OR, student simplifies the expression but does not show written calculations.
0 Points: No response or incorrect response.

Question 2:
2 Points: Student shows written work and simplifies the expression.
\[t(2-3x) \div 2x \rightarrow (-3)(2-3(6)) \div 2(6) = -3(-16) \div 12 = 48 \div 12 = 4\]
1 Point: Student substitutes numbers correctly but makes a small error in calculation. OR, student simplifies the expression but does not show written calculations.
0 Points: No response or incorrect response.

Question 3:
2 Points: Student shows written work and computes that they will need 4 boxes of flowers.
\[62 \text{ cupcakes} \times 5 \text{ flowers/cupcake} = 310 \text{ flowers}\]
\[310 \text{ flowers} \div 80 \text{ flowers/box} = 3.875 \rightarrow 4 \text{ boxes}\]
1 Point: Student shows correct process but makes a minor arithmetic error. OR, student gives correct response but does not show written steps
0 Points: No response or incorrect response.
Exercise #73

Standard: Number, Number Sense and Operations
Indicator: NNSO #7 (7th Grade)
Solve problems using the appropriate form of a rational number (fraction, decimal or percent).

1. Seven out of eight dentists recommend SparkleBright toothpaste.
   Thirteen out of fifteen dentists recommend CleanShine toothpaste. Which toothpaste is recommended by a higher percentage of dentists? Show your work.

2. Fareeda wants to buy a new dress. The dress she wants costs $28.00, but Fareeda has a coupon for a 15% discount. How much will the dress cost her? Show your work.
Scoring Guide

Point Value: 4

Question 1:
2 Points: Student converts each dentist recommendation into a percent. Using these percentages, the student explains that SparkleBright is recommended by 87.5% of dentists while CleanShine is recommended by 86.7% of dentists. Based on the percents, SparkleBright is the more recommended toothpaste.

1 Point: Student makes an error in calculating the percents, but gives the right answer based upon his or her calculations.

0 Points: No response or incorrect response.

Question 2:
2 Points: Student calculates that Fareeda’s dress will cost $23.80 on sale. Student shows calculations.

1 Point: Student makes a minor arithmetic error, OR student does not show work but gives the correct answer.

0 Points: No response or incorrect response
Exercise #74

Standard: Number, Number Sense and Operations

Indicator: NNSO #6 (7th Grade)
Simplify numerical expressions involving integers and use integers to solve real life problems.

Simplify the expression.

1. \(2p^2 - 3q\), when \(p = 4\) and \(q = -3\)

2. \(\frac{7x}{2y}\), when \(x = -12\) and \(y = 21\)

3. Each day, Loretta’s Bakery uses $220 for supplies and ingredients. On weekdays, the bakery sells an average of $360 worth of baked goods per day, and on the weekends they sell $260 worth of baked goods per day. Each week, what is the bakery’s profit? Show each step of your work.
Scoring Guide
Point Value: 6 (2 points apiece)

Question 1:
2 Points: Student shows written work and simplifies the expression.
\[ 2p^2 - 3q \rightarrow 2(16) - 3(-3) = 32 + 9 = 41 \]
1 Point: Student substitutes numbers correctly but makes a small error in calculation. OR, student simplifies the expression but does not show written calculations.
0 Points: No response or incorrect response.

Question 2:
2 Points: Student shows written work and simplifies the expression.
\[ \frac{7x}{2y} \rightarrow \frac{7(-12)}{2(21)} = \frac{-84}{42} = -2 \]
1 Point: Student substitutes numbers correctly but makes a small error in calculation. OR, student simplifies the expression but does not show written calculations.
0 Points: No response or incorrect response.

Question 3:
2 Points: Student shows written work and computes that the bakery’s profit will be $780 per week.
\[ 5($360) + 2($260) - 7($220) = $780 \]
1 Point: Student shows correct process but makes a minor arithmetic error. OR, student gives correct response but does not show written steps
0 Points: No response or incorrect response.
Monica wanted to know how well she was doing in her classes. Her math teacher said that she had $\frac{17}{20}$ possible points. Her science teacher said she had an 83% average. Her English teacher said she had 0.79 of the possible points for the class. Her social studies teacher said that she had 21 of the 25 possible points. List the classes in order from highest grade to lowest grade and show how you compared the grades.
Scoring Guide
Point Value: 2 Points

2 Points: A 2-point response contains the following elements:

- Student shows or explains converting the scores into a single comparable format:
  
  Math: 85%, 0.85, 85/100
  Science: 83%, 0.83, 83/100
  English: 79%, 0.79, 79/100
  S. Studies: 84%, 0.84, 84/100;

- Student lists the courses from highest average to lowest: math, social studies, science, and English.

**Note:** If students convert the numbers incorrectly, they should receive 1 point if they order the grades from highest to lowest according to their calculations.

1 Point: The response contains 1 of the 2 elements of a 2-point response

0 Points: The response contains none of the 2 elements of a 2-point response.
Exercise #76

Standard: Data Analysis and Probability
Indicator: DAP #7 (7th Grade)
Compute probabilities of compound events.

A bag contains 4 blue marbles and 2 green marbles. Steven draws one marble and then places it back in the bag. Then he draws another marble and places it back in the bag.

a. Show all of the possible outcomes for the two draws.

b. Are the events independent or dependent events? Explain how you know.

c. What is the probability that Steven will draw a blue marble first and a green marble second? Show your work.
Scoring Guide
Point Value: 4 Points

4 points: The response includes the following elements:

- The student correctly shows all possible outcomes—(Blue, Blue), (Blue, Green), (Green, Blue), (Green, Green);
- The student indicates that the events are independent because the marbles are replaced after each draw.
- The student shows that the probability of drawing a blue marble then a green marble is 2/9.
- The student must show how they got the correct answer.
  - Example: Probability (Blue) = 4/6 or 2/3
  - Probability (Green) = 2/6 or 1/3
  - Probability (Blue, Green) = \( \frac{2}{3} \times \frac{1}{3} = \frac{2}{9} \)

3 points The response includes 3 of the elements of a 4-point response.

2 points The response includes 2 of the elements of a 4-point response.

1 points The response includes 1 of the elements of a 4-point response.

0 points The response includes none of the elements of a 4-point response.

Teacher Notes:
Tree diagrams
The probability of drawing a blue marble on the first draw is equal to 4/6, the number of blue marbles in the bag, 4, divided by the total number of marbles in the bag, 6. Since marbles are replaced between draws, each draw is an independent event. The second draw is not dependent on the outcome of the first draw. The probability of drawing a green marble on the second draw is 2/6, the number of green marbles in the bag, 2, divided by the total number of marbles in the bag, 6. Therefore, the probability of drawing one blue marble then one green marble is 4/6 * 2/6 = 8/36 = 2/9.
Exercise #77

Standard: Data Analysis and Probability
Indicator: DAP #7 (7th Grade)
Compute probabilities of compound events.

Anthony’s school has a new uniform policy. He has a blue sweater, a white turtleneck, a white button-down shirt, and a blue polo shirt that he can wear to school.

1. If Anthony has one pair of blue pants and two pairs of tan pants in his closet, what is the probability that he will randomly select the following items?
   a. A Blue Shirt _______________________________
   b. Tan Pants ________________________________
   c. A Blue Shirt AND Tan Pants __________________

2. Anthony finds that if he randomly selects one pair of pants and one top, he has a $\frac{1}{4}$ probability that he will select a white top and a blue bottom. What combination of pants must he have in his closet if this probability is accurate?
   O A. two pairs of blue pants
   O B. one pair of blue pants and one pair of tan pants
   O C. two pairs of blue pants and one pair of tan pants
   O D. one pair of blue pants and three pairs of tan pants
Answer Key

Point Value: 4 Points (3 points + 1 point)

1. a. \( \frac{2}{4} = \frac{1}{2} \)

   b. \( \frac{2}{3} \)

   c. \( \frac{1}{2} \times \frac{2}{3} = \frac{2}{6} \) or \( \frac{1}{3} \)

2. B. one pair of blue pants and one pair of tan pants
Exercise #78

Standard: Data Analysis and Probability

Indicator: DAP #7 (7th Grade)
Compute probabilities of compound events.

The weather reporter says that there is a 50% chance of rain today and a 25% chance of rain tomorrow.

1. In the chart below, fill in the probabilities of “Rain” and “No Rain” for both days.

<table>
<thead>
<tr>
<th></th>
<th>P(Rain)</th>
<th>P(No Rain)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Today</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tomorrow</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. What is the probability that it will rain both today and tomorrow? Show your work.
Scoring Guide

Point Value: 6 Points (4 + 2)

1. (4 Points) One point for each correct entry.

<table>
<thead>
<tr>
<th></th>
<th>P(Rain)</th>
<th>P(No Rain)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Today</td>
<td>0.5 or 50%</td>
<td>0.5 or 50%</td>
</tr>
<tr>
<td>Tomorrow</td>
<td>0.25 or 25%</td>
<td>0.75 or 75%</td>
</tr>
</tbody>
</table>

2. 2 points: The response includes both of the following elements:
   - The response shows that the probability of rain on both days is 12.5% or 1/8 or 0.125;
   - Response indicates that P(Rain) for today and P(Rain) for tomorrow were multiplied to get the answer.
     - Example: $0.5 \times 0.25 = 0.125$

1 points: The response includes 1 of the elements of a 2-point response.

0 points: The response includes none of the elements of a 2-point response.
Exercise #79

Standard: Patterns, Function and Algebra

Indicator: PFA #10 (7th Grade)
Analyze linear and simple nonlinear relationships to explain how a change in one variable results in a change in another.

In the following graphs, time is represented on the x-axis and speed is represented on the y-axis. Using the relationship between time and speed for each graph, match each graph to the description that best represents it.

DESCRIPTIONS:
a. A stone rolls down a hill and then hits a wall.
b. You run down a hill and then walk at a steady pace on level ground.
c. You stop your bike at your friend’s house and then ride away after he gets on his bike.
d. You start at the top of a hill on your bike and let it coast down the hill, then ride it up another hill and then coast down that hill, then ride it up a bigger hill and stop at the top to catch your breath.
e. Your mom stops the car at a stoplight.

GRAPHS:

1)  
   \[ \text{Speed} \]
   \[ \text{Time} \]

2)  
   \[ \text{Speed} \]
   \[ \text{Time} \]

3)  
   \[ \text{Speed} \]
   \[ \text{Time} \]

4)  
   \[ \text{Speed} \]
   \[ \text{Time} \]

5)  
   \[ \text{Speed} \]
   \[ \text{Time} \]
Answer Key

Point Value: 5 Points

1. e
2. d
3. b
4. c
5. a
Exercise #80

Standard: Patterns, Function and Algebra

Indicator: PFA #11 (7th Grade)
Use graphing calculators or computers to analyze change; e.g., distance-time relationships.

On weekends, Pedro likes to participate in cycling tours around the state. Last weekend he averaged 10 miles per hour. How long did it take him to travel 150 miles? How much time would he have saved by increasing his rate by 5 miles per hour? Explain your answers and show all your work using words, numbers or symbols.
Scoring Guide

Point Value: 3 Points

3 points: Response indicates:

- It took Pedro 15 hours to travel 150 miles at 10 miles per hour and includes work or an explanation to support the response.
- It took Pedro 10 hours to travel 150 miles at 15 miles per hour and includes work or an explanation to support the response.
- Pedro would have saved 5 hours by increasing his rate of speed by 5 miles per hour and includes work or an explanation to support the response.

2 point: Response contains all elements of a 3-point response with an error in ONE of the bulleted score points.

1 point: Response contains all elements of a 3-point response with an error in TWO of the bulleted score points.

0 points: No response OR response contains THREE or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #81

Standard:  Patterns, Function and Algebra

Indicator:  PFA #4 (7th Grade)
Create visual representation of equation-solving processes that model the use of inverse operations.

Indicator:  PFA #7 (7th Grade)
Justify that two forms of an algebraic expression are equivalent, and recognize when an expression is simplified; e.g., \(4m = m + m + m + m\) or \(a \cdot 5 + 4 = 5a + 4\).

Simplify the algebraic equation \(10k - 3(k + 8) = 74\). Then solve for \(k\). Show the following in your work:

a. The process of combining like terms.
b. The new algebraic equation in simplest form.
c. The process of solving for \(k\) using inverse operations.
d. The solution for \(k\).
Scoring Guide

Point Value: 4 Points

4 points: Response indicates:

- An explanation of the process of distributing and combining like terms. Students should show that distributing \(-3\) over the \((k + 8)\) would result in \(10k - 3k - 24 = 74\). Then the variables are combined. (For example, \((10k - 3k) - 24 = 74\).)
- The resulting algebraic equation is \(7k - 24 = 74\).
- The additive inverse is used. (For example, \(7k - 24 + 24 = 74 + 24\).) The equation is simplified. (For example, \(7k = 98\).) Then the multiplicative inverse is used. (For example, \(\frac{7k}{7} = \frac{98}{7}\) OR \(\frac{1}{7} \cdot 7k = 98 \cdot \frac{1}{7}\).)
- The solution is \(k = 14\).

3 points: Response contains all elements of a 4-point response with an error in ONE of the bulleted score points.

2 points: Response contains all elements of a 4-point response with an error in TWO of the bulleted score points.

1 points: Response contains all elements of a 4-point response with an error in THREE of the bulleted score points.

0 points: No response OR response contains FOUR or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #82

Find the solution for the following equation: \(7x + 3 = 3(2x + 4)\). Show the following in your work:

a. The process of combining like terms.

b. The process of solving for \(x\) using inverse operations.

c. The solution for \(x\).
Scoring Guide
Point Value: 4 Points

4 points: Response indicates:

- An explanation of the process of distributing and combining like terms. Students should show that distributing 3 over the \((2x + 4)\) would result in \(7x + 3 = 6x + 12\).
- The additive inverse is used for \(6x\). (For example, \(7x - 6x + 3 = 6x - 6x + 12\)), the equation is simplified (For example, \(7x - 6x + 3 = 12\)) and like terms are combined resulting in the algebraic expression in simplest form (For example, \(x + 3 = 12\)).
- Then the additive inverse is used again. (For example, \(x + 3 - 3 = 12 - 3\).)
- The solution is \(x = 9\).

3 points: Response contains all elements of a 4-point response with an error in ONE of the bulleted score points.

2 points: Response contains all elements of a 4-point response with an error in TWO of the bulleted score points.

1 points: Response contains all elements of a 4-point response with an error in THREE of the bulleted score points.

0 points: No response OR response contains FOUR or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #83

Standard: Number, Number Sense and Operations
Indicator: NNSO#2 (7th Grade)

Explain the meaning of exponents that are negative or 0.

1. Which of the following expresses \( \frac{1}{81} \) in exponential form?

   O A. \(-9^2\)
   O B. \(9^{-2}\)
   O C. \(3^{-3}\)
   O D. \(-81^0\)

2. Which of the following is equivalent to \(5^0\)?

   O A. \(-5\)
   O B. \(0\)
   O C. \(1\)
   O D. \(5\)

3. Write each of the following as a whole number or fraction without exponents.

   a. \(5^{-3}\)

   b. \(x^0\) (x is an integer)
Answer Key

Point Value: 4 Points (1 point each)

1. B
2. C
3a. \[ \frac{1}{125} \]
3b. 1
Exercise #84

Standard: Number, Number Sense and Operations
Indicator: NNSO#2 (7th Grade)
Explain the meaning of exponents that are negative or 0.

Li’s teacher said that any number to the power of 0 equals 1. Li could not understand this rule. Her classmate Asan said he could show her that $4^0 = 1$. He wrote:

**Step 1:**

\[
\frac{4^3}{4^3} = 1
\]

**Step 2:**

\[
\frac{4^3}{4^3} = 4^{(3-3)} = 4^0
\]

**Step 3:**

Therefore, \( \frac{4^3}{4^3} = 4^0 = 1 \)

Is Asan’s logic correct? Support your answer by explaining whether each of Asan’s steps is correct or not.
Scoring Guide

Point Value: 4 Points

4 Points: The response includes the following 4 elements:

- A statement that says Asan’s logic is correct.
- Correct explanation for step 1 that says that any number divided by itself is 1.
- Correct explanation for step 2 that says when dividing two numbers with exponents, if the base numbers are the same, subtract the exponents.
- Correct explanation for step 3 that says that steps 1 and 2 were combined to prove that $4^0 = 1$.

3 Points: The response includes 3 of the elements of a 4-point response.

2 Points: The response includes 2 of the elements of a 4-point response.

1 Point: The response includes 1 of the elements of a 4-point response.

0 Points: The response shows no understanding of the concept.
Exercise #85

Standard: Number, Number Sense and Operations

Indicator: NNSO#2 (7th Grade)
Explain the meaning of exponents that are negative or 0.

Paula’s teacher asked her to prove that any number to the power of -1 equals 1 over that number. Paula said that she could show him that $2^{-1} = 1/2$. She wrote:

**Step 1:**
- $2^3 ÷ 2 = 2^2$
- $2^2 ÷ 2 = 2^1$
- $2^1 ÷ 2 = 2^0$
- $2^0 ÷ 2 = 2^{-1}$

**Step 2:**
$2^0 = 1$

**Step 3:**
$1 ÷ 2 = \frac{1}{2}$

**Step 4:**
Therefore, $2^{-1} = \frac{1}{2}$

Is Paula’s logic correct? Support your answer by explaining or proving whether each of Paula’s steps is correct or not.
Scoring Guide

Point Value: 4 Points

4 Points: The response includes the following 4 elements:

- A statement that her logic is correct.
- Correct explanation for step 1 that says if you divide $10^x$ by 10, you get $10^{x-1}$; OR when dividing two numbers with exponents, if the base numbers are the same, subtract the exponents.
- Correct explanation for step 2 that says that any number to the exponent 0 is 1.
- Correct explanation for step 4 that says that steps 2 and 3 were combined to prove that $10^0 = 1$.

3 Points: The response includes 3 of the elements of a 4-point response.

2 Points: The response includes 2 of the elements of a 4-point response.

1 Point: The response includes 1 of the elements of a 4-point response.

0 Points: The response shows no understanding of the concept.
Exercise #86

Standard: Number, Number Sense and Operations
Indicator: NNSO#3 (7th Grade)
Describe differences between rational and irrational numbers

Label each number as rational or irrational. Justify how you know which type of number it is.

<table>
<thead>
<tr>
<th>Number</th>
<th>Rational or Irrational?</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>\sqrt{100}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>\sqrt{1000}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>\frac{1}{3}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>\sqrt{2}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Scoring Guide**

Point Value: 6 Points (1 point each)

<table>
<thead>
<tr>
<th>Number</th>
<th>Rational or Irrational?</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.63</td>
<td>Rational</td>
<td>Is a decimal that terminates</td>
</tr>
<tr>
<td>$\sqrt{100}$</td>
<td>Rational</td>
<td>Integer OR can be written as a fraction, $\frac{10}{1}$</td>
</tr>
<tr>
<td>$\sqrt{1000}$</td>
<td>Irrational</td>
<td>Has a decimal equivalent that does not repeat or terminate</td>
</tr>
<tr>
<td>-15</td>
<td>Rational</td>
<td>Integer</td>
</tr>
<tr>
<td>$\frac{1}{3}$</td>
<td>Rational</td>
<td>Is a fraction</td>
</tr>
<tr>
<td>$\sqrt{2}$</td>
<td>Irrational</td>
<td>Has a decimal equivalent that does not repeat or terminate</td>
</tr>
</tbody>
</table>
Exercise #87

Standard: Number, Number Sense and Operations

Indicator: NNSO#3 (7th Grade)
Describe differences between rational and irrational numbers

Explain the difference between rational numbers and irrational numbers.

Give an example of a rational number: ________________

Give an example of an irrational number: ________________
Scoring Guide

Point Value: 4 Points

4 points: The response includes:

- A correct definition of a **rational** number – a rational number is a number that can be expressed as a ratio \( a/b \), where \( a \) and \( b \) are integers and \( b \neq 0 \);

- A correct definition of an **irrational** number – an irrational number cannot be written in the form \( a/b \), and has a decimal form that does not terminate or repeat;

- A correct example of a rational number – examples: a fraction, a repeating decimal, a terminating decimal, a whole number, or an integer (1/2, .333…, .23, 7, or -4, etc.);

- A correct example of an irrational number – examples: \( \pi, \sqrt{2}, \) etc.) NOTE: 3.14 is not a correct example of an irrational number (even if intention is to represent \( \pi \); 3.14… would be an acceptable way to represent \( \pi \).

3 Points: The response includes 3 of the elements of a 4-point response.

2 Points: The response includes 2 of the elements of a 4-point response.

1 Point: The response includes 1 of the elements of a 4-point response.

0 Points: The response shows no understanding of the concept.
Exercise #88

Standard: Number, Number Sense and Operations

Indicator: NNSO#1 (7th Grade)
Demonstrate an understanding of place value using powers of 10 and write large numbers in scientific notation.

1. How is the decimal 0.345 represented in expanded form?
   O a. \((3 + 0.1) \times (4 + 0.01) \times (5 + 0.001)\)
   O b. \((3 \times 0.01) \times (4 \times 0.001) \times (5 \times 0.0001)\)
   O c. \((3 + 1) + (4 + 0.1) + (5 + 0.01)\)
   O d. \((3 \times 0.1) + (4 \times 0.01) + (5 \times 0.001)\)

2. Write the decimal 0.66175 in expanded form.

3. How is this decimal 0.978 represented in expanded form using exponents?
   O a. \((9 \times 10^{-3}) + (7 \times 10^{-2}) + (8 \times 10^{-1})\)
   O b. \((9 \times 10^{-1}) + (7 \times 10^{-2}) + (8 \times 10^{-3})\)
   O c. \((9 \times 10^{2}) + (7 \times 10^{1}) + (8 \times 10^{0})\)
   O d. \((9 + 10^{-3}) \times (7 + 10^{-2}) \times (8 + 10^{-1})\)

4. Write the decimal 0.734821 in expanded form using exponents.
Answer Key

Point Value: 4 Points (1 point each)

1. D
2. $(6 \times 0.1) + (6 \times 0.01) + (1 \times 0.001) + (7 \times 0.0001) + (5 \times 0.00001)$
3. B
4. $(7 \times 10^{-1}) + (3 \times 10^{-2}) + (4 \times 10^{-3}) + (8 \times 10^{-4}) + (2 \times 10^{-5}) + (1 \times 10^{-6})$
Exercise #89

Standard: Number, Number Sense and Operations

Indicator: NNSO#1 (7th Grade)
Demonstrate an understanding of place value using powers of 10 and write large numbers in scientific notation.

1. Which number is written correctly in scientific notation?
   - a. $879 \times 10^5$
   - b. $87.9 \times 10^6$
   - c. $8.79 \times 10^7$
   - d. $0.879 \times 10^8$

2. The population of the African country, Kenya, is about 32,000,000. Write this number in scientific notation.

3. The diameter of the Sun is about $1.378 \times 10^6$ kilometers. Write this number in standard form.

4. The planet Venus has a diameter of 12,104 kilometers. Express this number in scientific notation.
Answer Key

Point Value: 4 Points (1 point each)

1. C
2. $3.2 \times 10^7$
3. 1,378,000
4. $1.2104 \times 10^4$

Teacher Notes:

Exponents are a shorthand way of representing repeated multiplication. When a number is written in exponential notation ($x^a$) there is a base ($x$) and an exponent ($a$). The exponent tells how many times the base is used as a factor, for example $4^3$ represents $4 \cdot 4 \cdot 4$.

The exponent can be positive ($2^6$), negative ($5^{-2}$), or zero ($4^0$). If the exponent is positive, then the base number is raised to that power. For example, $2^6$ represents $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$. If the exponent is negative, the reciprocal of the base number is raised to that power, for example, $5^{-2} = \frac{1}{5 \cdot 5}$. If the exponent is zero, this means the value is equal to 1 regardless of the value of the base. For example, $4^0 = 1$, $-6^0 = 1$; and $531^0 = 1$.

Scientific notation is a way of writing extremely small or large numbers in an easier way. A number is in scientific notation when it is written as the product of a number between one and ten multiplied by a power of ten (e.g. $3.2 \times 10^6$). When a number greater than one is written in scientific notation, the exponent of the power of ten is positive (e.g. $3,200,000 = 3.2 \times 10^6$). When a number smaller than one but greater than zero is written in scientific notation, the exponent of the power of ten is negative. (e.g. $0.00000046 = 4.6 \times 10^{-7}$).

An easy way to convert a number greater than one to scientific notation is to move the decimal point to the left until you get a number between one and ten. Each time you move the decimal point, you are dividing the number by ten. The number of places you move the decimal point represents the exponent for the power of ten.

For numbers between zero and one, the decimal point will be moved to the right to make a number between one and ten. Numbers between zero and one will have negative exponents.
Exercise #90

Standard: Number, Number Sense and Operations

Indicator: NNSO#1 (7th Grade)
Demonstrate an understanding of place value using powers of 10 and write large numbers in scientific notation.

A. In the year 2000, the population of the state of Ohio was estimated to be about $1.14 \times 10^7$.
   - Write the number for population in standard form.
   - Write the number for population in words.

B. If every Ohio citizen earned $10,000 that year, how much in total earnings would have been generated by the citizens of the state of Ohio in the year 2000?
   - Write your answer in standard form.
   - Write your answer in words.
   - Write your answer in scientific notation.
Answer Key

Point Value: 5 Points (1 point each)

a. 11,400,000; eleven million four hundred thousand
b. $114,000,000,000; one hundred fourteen billion dollars
c. $1.14 \times 10^{11}$

Teacher's Notes: A shortcut to solving parts B/C of the problem is by using the shortcut for multiplying exponential numbers that have the same base.

\[(1.14 \times 10^7) \times 10^4 = 1.14 \times (10^7 \times 10^4) = 1.14 \times 10^{7+4} = 1.14 \times 10^{11}\]

You may want to review the properties of exponents to make scientific notation even more useful:

- When multiplying two numbers with exponents, if the base numbers are the same, just add the exponents. For example, $10^5 \times 10^3 = 10^8$.
- When dividing two numbers with exponents, if the base numbers are the same, subtract the exponents. For example, $10^4/10^2 = 10^2$.

Students can discover what happens with exponential numbers when multiplying and dividing. Review rewriting a number raised to a power as a product of factors (e.g., $4^5 = 4 \times 4 \times 4 \times 4 \times 4$). Then provide students with a problem where two numbers with the same base but that have different exponents are multiplied (e.g., $4^5 \times 4^3$). Students rewrite the numbers as products of factors, combine the factors and rewrite as a single number raised to a power (e.g., $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^9$). After doing several problems, students should be able to verbalize the pattern and explain that when multiplying numbers with exponents that have the same base, the exponents are added and the base stays the same. The pattern for division with exponents can be found in a similar manner. Provide students with a division problem written in fraction form (e.g., $\frac{3^6}{3^4}$). Students rewrite the problem showing each number as a product of factors, and then simplify the fraction (e.g., $\frac{3^6}{3^4} = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3} = 3^2$. Students should be able to explain after doing several problems, that when numbers with the same base number and different exponents are divided, the exponents are subtracted. Be sure to include problems that will result in negative exponents, or include negative exponents in the problems, once students have an understanding of the pattern.
Last year, Charlene’s school and Roger’s school each had 475 students. This year, the number of students at Charlene’s school has increased by 35%. The number of students at Roger’s school is now 135% of last year’s population. Roger says that his school has more students this year than does Charlene’s school. Is he correct? Show calculations to justify your answer.
Scoring Guide
Point Value: 2

2 Points: Student explains that Roger is incorrect. The schools still have the same number of students because an increase of 35% is the same as 135% of the original. Student shows calculations to justify his or her answer.

1 Point: Student explains that Roger is incorrect and that the student populations will remain the same. The student does not justify his or her answer appropriately.

0 Points: No response or response that does not show understanding of the concept.
Exercise #92

Standard: Number, Number Sense and Operations

Indicator: NNSO#9 (7th Grade)
Represent and solve problem situations that can be modeled by and solved using concepts of absolute value, exponents, and square roots (for perfect squares).

1. $|35| - |78| = \underline{\hspace{2cm}}$

2. $35 + |-78| = \underline{\hspace{2cm}}$

3. On the number line below, show all solutions to the following inequality.

$|q| > 3$

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6
Scoring Guide
Point Value: 6 (1, 1, 4)

1. $|35| - |78| = -43$ (1 point for correct response)
2. $35 + |-78| = 113$ (1 point for correct response)
3. 
   4 Points: Response contains the following attributes:
   - Open circle at -3;
   - Open circle at +3;
   - Arrow extending from -3 toward $-\infty$;
   - Arrow extending from 3 toward $\infty$.

   3 Points: Response includes 3 of the elements from the complete response.
   2 Points: Response includes 2 of the elements from the complete response.
   1 Point: Response includes 1 of the elements from the complete response.
   0 Points: Response includes non elements from the complete response.
Exercise #93

Standard: Number, Number Sense and Operations

Indicator: NNSO#9 (7th Grade)
Represent and solve problem situations that can be modeled by and solved using concepts of absolute value, exponents, and square roots (for perfect squares).

Mr. Rashad’s Earth Science class is keeping track of the depth of a pond near their school. They made a table of how many centimeters the water level rose or fell each month.

<table>
<thead>
<tr>
<th>Month</th>
<th>Change in Water Level (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>September</td>
<td>2.5</td>
</tr>
<tr>
<td>October</td>
<td>-1.4</td>
</tr>
<tr>
<td>November</td>
<td>0.8</td>
</tr>
<tr>
<td>December</td>
<td>-2.3</td>
</tr>
<tr>
<td>January</td>
<td>-1.7</td>
</tr>
<tr>
<td>February</td>
<td>-1.0</td>
</tr>
<tr>
<td>March</td>
<td>-2.5</td>
</tr>
<tr>
<td>April</td>
<td>2.2</td>
</tr>
<tr>
<td>May</td>
<td>3.4</td>
</tr>
</tbody>
</table>

A. Calculate the total number of centimeters the water level rose and fell over the period from September to May. Show your work.

B. At the beginning of September, when the class began studying the pond, the water depth was 460 cm. Use the data from the table to calculate the depth of the pond in May, when the class took its last measurement.
Scoring Guide

Point Value: 4 Points

4 Points: Response contains the following:
- For Part A, response indicates that the water rose a total of 8.9 cm.
- For Part A, response indicates that the water fell a total of 8.9 cm.
- For Part B, student explains that the water rose and fell by the same amount.
- For Part B, response indicates that the level of the pond is the same at the end of the measurements as it was at the beginning;

3 Points: Response includes 3 of the elements from the complete response.

2 Points: Response includes 2 of the elements from the complete response.

1 Point: Response includes 1 of the elements from the complete response.

0 Points: Response includes non elements from the complete response.
Exercise #94

Standard: Number, Number Sense and Operations

Indicator: NNSO#9 (7th Grade)
Represent and solve problem situations that can be modeled by and solved using concepts of absolute value, exponents, and square roots (for perfect squares).

1. In a particular ant colony, the number of ants triples every day. On day 1, there are only three ants in the colony. Which of the expressions below would allow you to calculate the number of ants on day 7?

- a. $3 \times 7$
- b. $7^3$
- c. $3^7$
- d. $7 \times 7 \times 7$

2. Shaw saved up $500 and opened a Certificate of Deposit (CD) account at the bank that pays 5% interest every year. The bank manager gave him an equation to find out how much money he would have after a certain number of years. (Don’t forget to convert the rate % to a decimal!)

$$A = P \times (1 + x)^t$$

- $A =$ amount after $t$ years
- $P =$ principal (initial amount)
- $x =$ interest rate (as decimal)
- $t =$ number of years

In 4 years, what will be the value of Shaw’s Certificate of Deposit? Show your calculations.
Scoring Guide

Point Value: 3 points

Question 1:
1. c. $3^7$

Question 2:
2 Points: Student substitutes the numbers into the equation correctly and solves for A.
\[ A = P (1 + x)^t \]
\[ A = 500 (1 + 0.05)^4 \]
\[ A = 500 (1.05)^4 \]
\[ A = 607.75 \]

1 Point: Student substitutes the numbers correctly, but makes an error in calculation. OR Student makes a mistake in substituting numbers, but solves his or her equation correctly.

0 Points: No response. OR Response does not demonstrate understanding of the concept.
Exercise #95

Standard: Patterns, Function and Algebra
Indicator: PFA #3 (7th Grade)
Recognize and explain when numerical patterns are linear or nonlinear progressions; e.g., 1, 3, 5, 7 … is linear and 1, 3, 4, 8, 16 … is nonlinear.

The Fibonacci Sequence is a series of numbers that occur throughout both nature and art. They are found in the structure of crystals, the spiral of galaxies and the formation of a nautilus shell. The numbers in the sequence are:

0, 1, 1, 2, 3, 5, 8, 13 …

a. What is the next term in the sequence? How did you find it?
b. Is this a linear or nonlinear pattern? Explain your answer.
Scoring Guide

Point Value: 4 Points

4 points: Response indicates:

- The next term in the sequence is 21.
- The next term was found by adding the two previous terms.
- Identifies the sequence as nonlinear.
- Explains the reasoning for the sequence being nonlinear.
  Examples of explanations include but are not limited to: the sequence increases by a non-constant rate, OR the sequence would not create a straight line if graphed, OR the sequence increases by a different amount for each term, etc.

3 point: Response contains all elements of a 4-point response with an error in ONE of the bulleted score points.

2 point: Response contains all elements of a 4-point response with an error in TWO of the bulleted score points.

1 point: Response contains all elements of a 4-point response with an error in THREE of the bulleted score points.

0 points: No response OR response contains FOUR or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #96

Standard: Patterns, Function and Algebra

Indicator: PFA #3 (7th Grade)
Recognize and explain when numerical patterns are linear or nonlinear progressions; e.g., 1, 3, 5, 7 ... is linear and 1, 3, 4, 8, 16 ... is nonlinear.

The family tree of a honeybee follows the Fibonacci Sequence. A male honeybee has only one parent (a mother) because it develops from an unfertilized egg. A female honeybee has two parents (a mother and father) because it develops from a fertilized egg. The chart below shows the family tree of a male honeybee. The total number of honeybees for each generation is shown, starting with a male child.

<table>
<thead>
<tr>
<th>GENERATION</th>
<th>CHILD / PARENT GENDERS</th>
<th>TOTAL NUMBER of BEES at EACH GENERATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>MALE</td>
<td>1</td>
</tr>
<tr>
<td>Second</td>
<td>FEMALE</td>
<td>1</td>
</tr>
<tr>
<td>Third</td>
<td>MALE</td>
<td>FEMALE</td>
</tr>
<tr>
<td>Fourth</td>
<td>FEMALE</td>
<td>MALE</td>
</tr>
<tr>
<td>Fifth</td>
<td>...</td>
<td>... and so on.</td>
</tr>
</tbody>
</table>

... continued on next page.
a. What are the genders of each of the parents for the fifth generation?
b. How many total parents are there for the fifth generation?
c. How many total parents would there be for the tenth generation? Explain your answer.
Scoring Guide

Point Value: 4 Points

4 points: Response indicates:

- The genders of the fifth generation parents are male, female, female, male, female; OR two males and three females.
- There are five parents for the fifth generation.
- There are 55 parents for the tenth generation.
- The next term was found by adding the two previous terms.

3 points: Response contains all elements of a 4-point response with an error in ONE of the bulleted score points.

2 points: Response contains all elements of a 4-point response with an error in TWO of the bulleted score points.

1 point: Response contains all elements of a 4-point response with an error in THREE of the bulleted score points.

0 points: No response OR response contains FOUR or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #97

Standard: Patterns, Function and Algebra

Indicator: PFA #4 (7th Grade)
Create visual representation of equation-solving processes that model the use of inverse operations.

Manuel solved the equation: \( x - 2 = -6 \) on the number line below. He said the solution is \( x = -4 \). Is his solution correct? Explain how Manuel arrived at this solution.
Scoring Guide
Point Value: 2 Points

2 points: Response indicates:

- The solution $x = -4$ is correct.
- Since $x - 2$ is the same as $-6$, the bracket has a length of $-6$.
- To represent taking 2 away from $x$, an arrow is drawn to add 2.
  The arrow goes forward on the number line a distance of 2 to represent adding 2.

1 point: Response indicates:

- The solution $x = -4$ is correct, AND
- Contains an error in EITHER the second or third bulleted score points.

0 points: No response OR response contains TWO or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #98

Standard: Patterns, Function and Algebra

Indicator: PFA #4 (7th Grade)

Create visual representation of equation-solving processes that model the use of inverse operations.

Cassandra used Algebra Tiles to solve an equation. Her solution is pictured below.

a. What was the original equation?
b. What did she do in the first step?
c. What did she do in the second step?
d. What is her solution?

HINT: Remember that □ represents a variable, and □ represents a positive unit amount.
Scoring Guide
Point Value: 4 Points

4 points: Response indicates:

- The original equation is $3x + 4 = 10$.
- Explains that Cassandra first subtracted 4 from both sides of the equation OR added -3 to both sides of the equation, resulting in $3x = 6$.
- Explains that Cassandra then divided both sides of the equation by 3 OR multiplied both sides of the equation by $\frac{1}{3}$, resulting in $x = 2$.
- The solution is $x = 2$.

3 points: Response contains all elements of a 4-point response with an error in ONE of the bulleted score points.

2 points: Response contains all elements of a 4-point response with an error in TWO of the bulleted score points.

1 point: Response contains all elements of a 4-point response with an error in THREE of the bulleted score points.

0 points: No response OR response contains FOUR or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #99

Standard: Patterns, Function and Algebra

Indicator: PFA #4 (7th Grade)
Create visual representation of equation-solving processes that model the use of inverse operations.

Using Algebra Tile models, draw an illustration that represents solving the equation – 6x + 1 = 7 using inverse operations. Record your steps in both picture and number format.

HINT: Remember that □ represents a positive variable x, □ represents a positive unit amount, □ represents a negative variable x and □ represents a negative unit amount.
### Scoring Guide

Point Value: 4 Points

<table>
<thead>
<tr>
<th>Process</th>
<th>Picture Format</th>
<th>Number Format</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Original Equation</strong></td>
<td>![Original Equation Diagram]</td>
<td>– 6x + 1 = 7</td>
</tr>
<tr>
<td><strong>Additive Inverse</strong></td>
<td>![Additive Inverse Diagram]</td>
<td>– 6x + 1 + – 1 = 7 + – 1 OR – 6x + 1 – 1 = 7 – 1</td>
</tr>
<tr>
<td><strong>Simplified Equation</strong></td>
<td>![Simplified Equation Diagram]</td>
<td>– 6x = 6</td>
</tr>
<tr>
<td><strong>Multiplicative Inverse</strong></td>
<td>![Multiplicative Inverse Diagram]</td>
<td>– 6x • – (\frac{1}{6}) = 6 • – (\frac{1}{6}) OR (\frac{-6x}{-6}) = (\frac{6}{-6})</td>
</tr>
<tr>
<td><strong>Solution</strong></td>
<td>![Solution Diagram]</td>
<td>x = – 1</td>
</tr>
</tbody>
</table>

*NOTE: Students may not have shown the simplified equation in the third row. However, they must demonstrate the additive and multiplicative inverses.*

3 points: Response contains all elements of a 4-point response with an error in ONE of the boldface score points in the chart.

2 points: Response contains all elements of a 4-point response with an error in TWO of the boldface score points in the chart.

1 point: Response contains all elements of a 4-point response with an error in THREE of the boldface score points in the chart.

0 points: No response OR response contains FOUR errors or more errors in the boldface score points in the chart OR demonstrates no understanding of concept.
Exercise #100

Standard: Patterns, Function and Algebra

Indicator: PFA #4 (7th Grade)
Create visual representation of equation-solving processes that model the use of inverse operations.

When Jackie solved the equation $\frac{x}{2} - 3 = 5$, she wrote down her solution as $x = 4$.

Diane said that answer was wrong, and that $x = 16$. One of the students is correct and one is incorrect. Which student had the correct answer? Explain why the other student’s answer was incorrect.
Scoring Guide
Point Value: 2 Points

2 points: Response indicates:

- Diane has the correct answer.
- An explanation that tells why Jackie’s answer is incorrect. (For example, because although Jackie correctly added 3, she then divided by 2, when she should have multiplied by 2; OR Jackie’s answer is incorrect because if you replace 4 for $x$ in the equation and then solve, you get $\frac{4}{2} - 3 \neq 5$, which is $2 - 3 = -1$; OR Diane’s answer is correct because if you replace 16 for $x$ in the equation and then solve, you get $\frac{16}{2} - 3 = 5$, which is $8 - 3 = 5$.)

1 point: Response contains all elements of a 2-point response with an error in ONE of the bulleted score points.

0 points: No response OR response contains TWO or more errors in the bulleted score points OR demonstrates no understanding of concept.
Saturn, the sixth planet from the sun, has a diameter that is about 10 times larger than that of the Earth, the third planet from the sun.

a. How many times larger is Saturn’s surface area than that of the Earth? Show work or provide an explanation for your answer.
   *(Hint: The surface area of a sphere is $4\pi r^2$.)

b. How many times larger is Saturn’s volume than that of the Earth? Show work or provide an explanation for your answer.
   *(Hint: The volume of a sphere is $\frac{4}{3}\pi r^3$.)
**Scoring Guide**

Point Value: 4

Part A:
2 Points: A complete response contains the following.
- Student responds that the surface area of Saturn is 100 times larger than is the surface area of the Earth.
- The student may either show written work or provide an explanation in order to justify his or her response.

1 Point: Response includes 1 of 2 parts of the complete response.
0 Points: Response includes 0 of 2 parts of the complete response.

Part B:
2 Points: A complete response contains the following.
- Student responds that the volume of Saturn is 1000 times larger than is the volume of the Earth.
- The student may either show written work or provide an explanation in order to justify his or her response.

1 Point: Response includes 1 of 2 parts of the complete response.
0 Points: Response includes 0 of 2 parts of the complete response.
Exercise #102

Standard: Geometry and Spatial Sense

Indicator: Geometry #1 (Grade 7)
Use proportional reasoning to describe and express relationships between parts and attributes of similar and congruent figures.

Figures WXYZ and NPQR are both quadrilaterals. Casey thinks that the figures are congruent because the measure of each of the four sides of both figures is 10 cm.

Is Casey correct in thinking that the figures must be congruent? Support your argument using your knowledge of similar and congruent figures.
Scoring Guide
Point Value: 2

2 Points: Student response reflects either of the following sets of bulleted points.

- Response explains that Casey is incorrect in thinking that the quadrilaterals must be congruent.
- Student justifies that the two figures might be different. Possible explanations include: the angle measures might be different; OR the figures could be rhombuses or squares; OR Without knowing the angles, it is not possible to tell whether the quadrilaterals are congruent.

OR

- Student explains that the two quadrilaterals are not necessarily congruent.
- Student justifies that the two figures might be different. Possible explanations include: the angle measures might be different; OR the figures could be rhombuses or squares; OR Without knowing the angles, it is not possible to tell whether the quadrilaterals are congruent.

1 Point: Response includes one of the two parts of the 2-point response.

0 Points: Response does not include either of the two parts of the 2-point response.

Teacher Note: In order to answer this question, students need to think about what makes two figures similar, congruent or neither. Students might be tempted immediately to think that the two quadrilaterals in this item are congruent because the side lengths are the same. Ask them to think of another shape that has equal side lengths, such as a rhombus.
Exercise #103

Standard: Geometry and Spatial Sense

Indicator: Geometry #4 (Grade 7)
Determine necessary conditions for congruence of triangles.

Given: \( \overline{FH} \cong \overline{JL} \)
\( \angle GHF \cong \angle KLJ \)

The information above is not sufficient to prove that triangles \( FGH \) and \( JKL \) are congruent. With three pieces of information it is possible to tell whether or not the triangles are congruent.

Below, give two pieces of information, each of which could serve as the third piece of information to prove that the triangles are congruent. In each case, state the property of triangles that is satisfied.
Scoring Guide

Point Value: 2

2 Points: Response contains the following information: two separate attributes of the triangles that would allow one to prove that the triangles are congruent:

1. $\angle HFG \cong \angle LJK$
2. $\angle FGH \cong \angle JKL$
3. $\overline{GH} \cong \overline{KL}$

1 Point: Response provides one correct attribute of the triangles that would allow one to prove that the triangles are congruent.

0 Point: No correct response provided.
Exercise #104

Standard: Geometry and Spatial Sense

Indicator: Geometry #6 (Grade 7)
Determine and use scale factors for similar figures to solve problems using proportional reasoning.

1. James has a model car collection. One of his model trucks is measures ½ foot in length. The real truck measures 14 feet in length. What is the scale of the model truck to the real truck?

- a. 1 : 2
- b. 1 : 7
- c. 1 : 14
- d. 1 : 28

2. The blueprint for Tamika’s new house uses a scale of ½ inch equals 1 foot.

The dimensions of the dining room on the blueprint are 4 inches by 8 inches. Calculate the actual dimensions of Tamika’s dining room. Show how you found the length and width. Make sure to include the appropriate units of measure.
Answer Key/Scoring Guide

Point Value: 3 (2 points + 1 point)

1. D 1 : 28

2. See Guidelines:

2 Points: A complete response contains the following elements.

- Student calculates that the dimensions of the dining room are 16 feet by 8 feet.
- Written work shows how the student used the scale to calculate the new dimensions or includes a description of how to use the scale.

1 Point: Response includes 1 of 2 elements of the complete response.

0 Points: Response includes 0 of 2 elements of the complete response.
Exercise #105

Standard: Measurement
Indicator: Measurement #1 (Grade 7)
Select appropriate units for measuring derived measurements.

Tony calculated two different measurements for his car's gas mileage. The first measurement had units of “miles per gallon” and the second measurement had units of “gallons per mile.” In each case, describe what each measurement tells you about the gas mileage of Tony’s car.

Measurement 1:

Measurement 2:
Scoring Guide

Point Value: 2 Points

2 Points: A complete response contains the two following elements.
  
  • Explanation that the “miles per gallon” measurement tells how many miles the car can go on a particular amount of gasoline.
  
  • Explanation that the “gallons per mile” measurement tells how many gallons of gas it takes to drive a particular distance.

1 Point: Response includes 1 of 2 elements of the complete response.

0 Points: Response includes 0 of 2 elements of the complete response.
Exercise #106

Standard: Measurement

Indicator: Measurement #2 (Grade 7)
Convert units of area and volume within the same measurement system using proportional reasoning and a reference table when appropriate.

The volume of a swimming pool is 6 m$^3$. What is the volume of the swimming pool in cubic centimeters (cm$^3$)? Show your work using words, numbers, or symbols.
Scoring Guide
Point Value: 3

3 Points: A complete response includes the following elements.

- Student work shows the conversion from cubic meters to cubic centimeters using any combination of numbers, words, or symbols. For example, a response might include an explanation that the number of cubic centimeters is 1,000,000 times of the number of cubic meters; OR explanation that ratio of cubic meters to cubic centimeters is 1:1,000,000 (or $1^3$ to $10^3$); OR calculations.

$$\frac{1,000,000 \text{ cm}^3}{1 \text{ m}^3} = \frac{x \text{ cm}^3}{6 \text{ m}^3}$$

$$x = 6,000,000 \text{ cm}^3$$

- Student gives the correct answer—the volume of the pool is 6,000,000 cm$^3$.

- Student provides the correct label with the numerical answers (cm$^3$).

2 Points: Student work contains 2 of 3 elements of the complete response.

1 Point: Student work contains 1 of 3 elements of the complete response.

0 Points: Student work contains 0 of 3 elements of the complete response.
Exercise #107

Standard: Measurement

Indicator: Measurement #4 (Grade 7)
Solve problems involving proportional relationships and scale factors.

Michelle walks to Beauregard Middle School each day. She looked on a map to figure out how many miles her school is from her house. On the map, the distance was $1\frac{1}{8}$ inches. The scale of the map says that $\frac{1}{2}$ inch equals 1 mile. How far is Michelle’s walk to school? Show your work by using numbers, words or symbols.
Scoring Guide

Point Value: 3 points

1. 3 Points: Student response includes these elements.
   - Student response shows use of the given scale to find the distance by using any combination of numbers, words, or symbols. For example, the student explains that the number of miles will be two times the number of inches shown on the map. Or, student might include calculations.

\[
\frac{\frac{1}{2} \text{ inch}}{1 \text{ mile}} = \frac{9/8 \text{ inches}}{x \text{ miles}} = \frac{1 \text{ inch}}{2 \text{ miles}} = \frac{9 \text{ inches}}{8 \text{ miles}} \quad x = 18/8 \text{ or } 2\frac{1}{4} \text{ miles}
\]

   - Student gives the correct distance of 2\frac{1}{4} miles.
   - Numerical answers have the correct labels.

2 Points: Student work contains 2 of 3 elements of the complete response.

1 Point: Student work contains 1 of 3 elements of the complete response.

0 Points: Student work contains 0 of 3 elements of the complete response.
Exercise #108

Standard: Geometry and Spatial Sense

Indicator: Geometry #2 (Grade 7)
Determine sufficient (not necessarily minimal) properties that define a specific two-dimensional figure or three-dimensional object.

Figures 1, 2, and 3 all follow the three properties listed below.

1. This shape is a quadrilateral.
2. This shape has two pairs of congruent angles.
3. This shape has two pairs of congruent sides.

a. Add a fourth property so that only Figure 2 and Figure 3 match all four properties.

b. Add a fourth property so that only Figure 3 matches all four properties.
Scoring Guide

Point Value: 2

2 Points: Response contains the following information:

- Response indicates a property so that only Figures 2 and 3 fit the properties. For example, “This shape has only right angles” or “All four of the angles in this figure are congruent.”
- Response includes a property that will exclude all figures except for Figure 3. For example, “This shape has four congruent sides.” The student may include more information, such as, “This shape has four congruent sides and four congruent angles.”

1 Point: Response contains 1 of the 2 parts of the 2-point response.

0 Points: Response contains 0 of the 2 parts of the 2-point response.

Teacher Note: This item illustrates how properties can be created that define a specific set of geometric shapes. By adding and subtracting properties, we can narrow or widen the set of shapes we define.

From this item, students can see that one type of figure can be part of the set of other types of figures. For instance, the square is part of the set of rectangles, the rectangle is part of the set of parallelograms, and the square is part of the set of parallelograms.
Exercise #109

Standard: Patterns, Function and Algebra

Indicator: PFA #8 (7th Grade)
Use formulas in problem solving situations.

Five years ago Ms. Hayes deposited $7,000 in a savings account and left it to earn interest. She received a statement from her bank showing she had earned interest on the money she had left in the savings account at a rate of 8% per year. Using the formula for simple interest, \( I = prt \) (where \( I \) is the interest, \( p \) is the principal, \( r \) is the rate, and \( t \) is the time in years), find the interest (\( I \)) accrued over the 5 years. Show work to support your answer or explain your answer.
Scoring Guide

Point Value: 2 Points

2-point Response indicates:

- The interest is $2,800.
- Clearly shows or explains how the formula was used to find the interest. For example, but not limited to:
  The student shows:
  \[ I = 7,000 \cdot 0.08 \cdot 5 \text{ years}; I = 560 \cdot 5; I = 2,800 \]

  OR Explains that they multiplied $7,000 by 0.08 (the equivalent for 8%), and then multiplied by 5 years.

1-point Response contains all elements of a 2 point response with an error in ONE of the bulleted score points.

0-points No response OR response contains TWO more errors in the bulleted score points OR demonstrates no understanding of the concept.
Exercise #110

Standard: Geometry and Spatial Sense

Indicator: Geometry #8 (Grade 7)  
Perform translations, reflections, rotations and dilations of two-dimensional figures using a variety of methods.

Jack started with the grey figure above. After performing two geometric transformations, he ended up with the black figure. Using specific numbers and geometry terms, describe how Jack may have transformed the original figure.
Scoring Guide

Point Value: 2

2 Points: Student explains one of two sequences of steps that Jack could have performed.

(1) Jack reflected the shape across the y-axis and translated the shape downward 8 units (or 8 in the negative y direction or -8 in the y direction).

(2) Jack reflected the shape across the x-axis and translated the shape 12 units to the right (or 12 in the positive x direction or +12 in the x direction).

(NOTE: Within choices (1) and (2) the order of the steps does not matter.)

1 Point: Student provides a basic description of what Jack did, but he or she does not use specifics. For example, the student might not include the number of units Jack translated the figure, or the student might not use the appropriate geometry terms (e.g. uses “flipped” for “reflected” or “moved” for “translated”).

0 Points: Student response reflects no understanding of the processes that Jack used.
Exercise #111

Standard: Geometry and Spatial Sense
Indicator: Geometry #8 (Grade 7)
Perform translations, reflections, rotations and dilations of two-dimensional figures using a variety of methods.

The vertices of triangle $FGH$ are located at the coordinates below.

$$F(3,2), G(10,5), H(7,-1)$$

a. Draw and label triangle $FGH$.

b. Rotate the triangle by 180°. Use the origin as the center of rotation. Label the vertices of triangle $F'G'H'$.
Scoring Guide
Point Value: 4

4 Points: A complete response contains the following elements:
   • Student draws triangle \( FGH \) (see example)
   • Student draws the triangle rotated 180 degrees about the origin—
     points are at \( F'(-3,-2); \ G'(-10,-5); \ H'(-7,1) \).
   • Student labels \( FGH \) (see example)
   • Student labels \( F'G'H' \) (see example).

3 Points: Response includes 3 of 4 elements of the complete response.
2 Points: Response includes 2 of 4 elements of the complete response.
1 Point: Response includes 1 of 4 elements of the complete response.
0 Points: Response includes 0 of 4 elements of the complete response.
**Teacher Note:** Students might get confused by the concept of rotating an object about the origin. Instead of performing the proper rotation, they might rotate a figure using the figure’s center as the center of rotation. They might find a diagram helpful for understanding this type of rotation, such as the one given here. Here, points G and G’ are positioned 180 degrees from one another. However, the triangle was not simply rotated in place. Rather, the points are rotated around an imaginary circle with a center at the origin and passing through both points G and G’.
A square pyramid with side length 4 cm and height 4 cm sits on its base inside a cube with side length 4 cm.

1. Make a sketch of the paired objects and label the dimensions.
2. How will the pair of figures look when they are viewed from the top? Draw and label a sketch of this top-view perspective.
3. How will the pair of figures look when they are viewed from the side? Draw and label a sketch of this side-view perspective.
Scoring Guide

Point Value: 4

4 Points: A complete response contains the following elements:

- Student draws the three-dimensional view properly. (See below)
- Student labels the dimensions of the three-dimensional view (See below)
- Student draws a front view of the pyramid inside the cube.
- Student draws a top view of the pyramid inside the cube.

3 Points: Response includes 3 of 4 elements of the complete response.

2 Points: Response includes 2 of 4 elements of the complete response.

1 Point: Response includes 1 of 4 elements of the complete response.

0 Points: Response includes 0 of 4 elements of the complete response.
Exercise #113

A group of friends tested themselves to see how many times each person could hit a tennis ball against the wall without missing. The results are below:

8  25  10  31  14  2  24  19  3  16  22  7  14  5  21  3

1. What is the range of the data?
   O A.  2
   O B.  16
   O C.  29
   O D.  31

2. If the friends decided to report their performance by arranging the results from least to greatest and choosing the middle score, which of the following will they report?
   O A.  the mean
   O B.  the median
   O C.  the mode
   O D.  the lower quartile

3. If the friends hired a tennis instructor to teach the four people with the fewest hits, which of the following would help them figure out who would get the lessons?
   O A.  the mode
   O B.  the median
   O C.  the lower extreme
   O D.  the lower quartile

Standard:  Data Analysis and Probability
Indicator:  DAP #3 (7th Grade)
Analyze a set of data by using and comparing combinations of measures of center (mean, mode, median) and measures of spread (range, quartile, interquartile range).
Answer Key

Point Values: 3 Points

1. C
2. B
3. D
The eight houses on Juniper Drive were each worth $125,000. Two new houses were built that were each worth $250,000. Describe how the value of the new houses will affect the mean and median house values for Juniper Drive. Use mathematics to support your answer.
Scoring Guide

Point Value: 2 Points

2 points: The response includes an adequate description of the effect of the new house’s value on the mean and median house values. The response also includes mean and median calculations to support the response.

- Old mean: $125,000; New mean = ((8\times125,000) + (2\times250,000))/10 = $150,000; the mean will increase.
- Old median: $125,000; New median: $125,000 is still the middle value; the median stays the same.

1 point: The response provides partial evidence of analyzing a set of data by using and comparing the mean and median as well as describing how the inclusion of outliers affects those measures; however, the solution is incomplete or slightly flawed.

For example, the response may
- Describe the effect of the new house’s value but provide no mathematics to support the answer.
- Describe the effect of the new house’s value on only the mean or only the median, with supporting work/explanation.
- Calculate the old and new mean and median but not describe the effect on house values.

0 points: The response is incorrect or irrelevant.

Teacher’s Note:

The goal of this item is to assess whether a student recognizes that mean and median function differently when an outlier(s) is added into a data set. In this situation, the new house worth $250,000 pulls up the “average” value of the houses in the neighborhood when you use the mean to describe the “average”; it changes from $125,000 to $150,000. However, if you use the median as the “average” to describe the data set, the “average” house value on Juniper stays the same ($125,000) with the addition of an outlier(s) (the new much higher valued house). If someone were to need to know the average house value on Juniper Drive, the median would give a more accurate representation of the house values.

Students should see that certain measures of central tendency are more appropriate to use than others in a given situation. For example, for sets of data with no outliers (points much higher or lower than most of the others) the mean usually works well. For data sets with a couple of points much higher or lower than most of the others, the median may be a good choice. For sets of data with many identical data points, the mode may be the best description for the data set.
Exercise #115

Standard: Data Analysis and Probability

Indicator: DAP #4 (7th Grade)
Construct opposing arguments based on analysis of the same data, using different graphical representations.

The Tourism Office in Detroit, Michigan, is publishing a booklet for visitors to the city. Qian and Robert each made a graph for the section called “What’s the Weather like in Detroit?” The publisher needs to decide which graph to use.

Qian’s Graph:

Average Monthly High Temperature (°F) in Detroit, Michigan

Robert’s Graph:

Average Yearly High Temperature (°F) in Detroit, Michigan
a. If the Tourism Office uses Qian’s graph, what conclusion might visitors make about the temperature in Detroit?

b. If the Tourism Office uses Robert’s graph, what conclusion might visitors make about the temperature in Detroit?

c. Which graph do you think the Tourism Office should use? Explain.
Scoring Guide

Point Value: 4 Points

4 Points: The response has all 4 of the following elements.
- The response indicates that, with Qian’s graph, visitors would conclude that the high temperature ranges from 30s-40s in the winter to 70s-80s during the summer.
- The response indicates that, with Graph 2, readers might conclude that the temperature in Detroit is usually around 55 to 60 degrees.
- The response indicates that the Tourism Office should use Qian’s graph.
- The response includes a reasonable explanation of why the newspaper should use Qian’s graph.
  - Example 1: Qian’s graph gives visitors more detail about the differences in temperature in Detroit over the course of the year.
  - Example 2: If the Tourism Office uses Robert’s graph, visitors might be misled to think that the temperature is more consistent throughout the year than it actually is.

3 Points: The response has 3 elements of a 4-point response.

2 Points: The response has 2 elements of a 4-point response.

1 Point: The response has 1 element of a 4-point response

0 Points: The response has none of the elements of a 4-point response.
Exercise #116

Standard: **Patterns, Function and Algebra**

Indicator: PFA #7 (7th Grade)

*Justify that two forms of an algebraic expression are equivalent, and recognize when an expression is simplified; e.g., \(4m = m + m + m + m\) or \(a \cdot 5 + 4 = 5a + 4\).*

Neela used Algebra Tiles to show how she simplified an algebraic equation so she could solve it. Her explanation is as follows:

<table>
<thead>
<tr>
<th>Original Picture Form</th>
<th>The original equation is (5x - 2x = 2 - 8).</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Combining Process</th>
<th>On the left side of the equation, two of the positive (x)'s and two of the negative (x)'s form zero pairs and cancel each other out. That gives you (3x). On the right side of the equation, two of the positive units and two of the negative units form zero pairs and cancel each other out. That gives you (-6).</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Result:</th>
<th>The result is (3x = -6).</th>
</tr>
</thead>
</table>

... continued on next page.
Using Neela’s explanation as an example for the problem below:

a. tell what the original equation is.

b. explain how the terms were combined to simplify the equation.

c. give the simplified equation before it is solved for x.
Scoring Guide

Point Value: 6 Points

6 points: Response indicates:

- Gives the original equation as \(3x + 2 - x = -2x - 6\) (or an equivalent equation).
- Two positive \(x\)’s were added to both sides of the equation.
- The positive \(x\)’s and negative \(x\)’s formed zero pairs on the both sides of the equation and canceled each other out.
- Two negative units were added to each side of the equation.
- The positive units and negative units formed zero pairs on the left side of the equation and canceled each other out.
- The resulting simplified equation is \(4x = -8\).

5 points: Response contains all elements of a 6-point response with an error in ONE of the bulleted score points.

4 points: Response contains all elements of a 6-point response with an error in TWO of the bulleted score points.

3 points: Response contains all elements of a 6-point response with an error in THREE of the bulleted score points.

2 points: Response contains all elements of a 6-point response with an error in FOUR of the bulleted score points.

1 points: Response contains all elements of a 6-point response with an error in FIVE of the bulleted score points.

0 points: No response OR response contains SIX or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #117

Standard: Patterns, Function and Algebra

Indicator: PFA #7 (7th Grade)
Justify that two forms of an algebraic expression are equivalent, and recognize when an expression is simplified; e.g., $4m = m + m + m + m$ or $a \cdot 5 + 4 = 5a + 4$.

Marie used Algebra Tiles to simplify an algebraic expression. Her work is pictured below. For Marie’s work:

a. Give the original algebraic expression.

b. Explain Marie’s combining process using words, numbers or symbols.

c. Give the resulting algebraic expression in simplest form.

<table>
<thead>
<tr>
<th>Original Picture Form</th>
<th>Combining Process</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Original Picture Form" /></td>
<td><img src="image2" alt="Combining Process" /></td>
<td><img src="image3" alt="Result" /></td>
</tr>
</tbody>
</table>
Scoring Guide

Point Value: 3 Points

3 points: Response indicates:

- The original algebraic expression is $5x + 4 - 3x + 5$ (or an equivalent expression).
- An explanation of the process of combining like terms. (For example, when the variables and units are combined, three positive and negative $x$’s cancel out (Zero Pairs) leaving 2 $x$’s, and the positive units combine, OR $(5x - 3x) + (4 + 5)$, etc.)
- The resulting algebraic expression is $2x + 9$.

2 points: Response contains all elements of a 3-point response with an error in ONE of the bulleted score points.

1 points: Response contains all elements of a 3-point response with an error in TWO of the bulleted score points.

0 points: No response OR response contains THREE or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #118

Standard: Patterns, Function and Algebra

Indicator: PFA #6 (7th Grade)
Represent inequalities on a number line or a coordinate plane.

The price for a concert ticket is more than $30. Write an inequality to describe the situation and then draw a number line and graph the inequality.
Scoring Guide

Point Value: 2 Points

2 points: Response indicates:

- An appropriate inequality for the problem. For example, but not limited to, equivalent inequalities, such as:
  \[ t > 30 \quad \text{OR} \quad 30 < t \]
- A number line drawn, and the inequality correctly graphed. For example:

1 points: Response contains all elements of a 2-point response with an error in ONE of the bulleted score points.

0 points: No response OR response contains TWO or more errors in the bulleted score points OR demonstrates no understanding of concept.
Exercise #119

Standard: Data Analysis and Probability

Indicator: DAP #1 (7th Grade)
Read, create and interpret box-and-whisker plots, stem-and-leaf plots, and other types of graphs.

James and Sara want to get new cell phones. The salesperson shows them a graph of the monthly costs of three possible service plans.

James will always use less than 100 minutes each month. Which plan should he choose? Explain your answer by comparing the costs of the 3 plans.

Sara will always use more than 300 minutes per month. Which plan should she choose? Explain your answer by comparing the costs of the 3 plans.
Scoring Guide

Point Value: 4 Points (2 points each)

Part 1:

2 Points: The response identifies Plan A as the best plan for James.
- The response includes an explanation that Plan A is the cheapest plan at 100 minutes or less.
- The response includes an explanation that at 100 minutes or less, Plan A is $30, Plan B is $40, and Plan C is $50.

1 Point: The response identifies Plan A as the best plan for James but gives an incomplete or no explanation.

0 Points: The response identifies an incorrect plan as the best plan for James.

Part 2:

2 Points: The response identifies Plan C as the best plan for Sara.
- The response identifies Plan C as the best plan because it is the cheapest plan at 320 minutes (student may estimate a value close to 320) and above.
- The response includes an explanation that although Plan B is the cheapest plan at exactly 300 minutes, Sara knows that she will always go over 300 minutes.

Note: It is possible to give full credit for a response that says that Sara should select plan B IF the response includes an explanation that between 300 and 320 minutes (student may estimate a value close to 320) plan B is cheapest (OR that Sara has to stay under 320 minutes a month).

1 Point: The response identifies Plan C but gives an incomplete or no explanation.

0 Points: The response identifies Plan B with no explanation or Plan A as the best plan for Sara.
Exercise #120

In Janine’s exercise class, the teacher marked down the number of sit ups each person could do without stopping.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Janine</td>
<td>41</td>
<td>Maria</td>
<td>46</td>
<td>Deena</td>
<td>66</td>
</tr>
<tr>
<td>Camille</td>
<td>51</td>
<td>Donia</td>
<td>74</td>
<td>Jenni</td>
<td>75</td>
</tr>
<tr>
<td>Wendy</td>
<td>23</td>
<td>Ciera</td>
<td>20</td>
<td>Lisa</td>
<td>28</td>
</tr>
</tbody>
</table>

Create a stem-and-leaf plot of the data and give the plot an appropriate title.
Scoring Guide

Point Value: 4 Points

4 Points: The student draws a correct stem and leaf plot.
- The stem includes consecutive numbers from 1 through 7 (including the 3);
- The leaves are in ascending numerical order;
- All data is represented (including 2 leaves (1’s) which represent the two 41’s);
- A title is present.

3 Points: The student draws a stem and leaf plot, including 3 of the elements of a 4-point response.

2 Points: The student draws a stem and leaf plot, including 2 of the elements of a 4-point response.

1 Point: The student draws a stem and leaf plot, including 1 of the elements of a 4-point response.

0 Points: The student does not understand the concept of a stem and leaf plot.

Teacher Notes: A Stem and Leaf Plot is a useful method for grouping and organizing data. The “leaves” consist of the numbers on the right side of the plot. The ones digit of each item represented in the plot makes up each individual leaf. These leaves are organized by “stems,” or the remaining shared digits from the individual numbers.

The stems are always organized vertically. All categories between the minimum and the maximum value are included, even if there are no leaves to fit those categories. For example, in the plot above, there are no leaves to put in the category ranging from 30-39. Within a category, leaves are ordered numerically. When a number appears more than once in a set of data, each appearance must be represented in the plot. Notice above that there are two 1’s each representing one occurrence of the number 41.

Stem and Leaf Plots are particularly useful because they show us the distribution of values in a set of data. Students can think of them as histograms turned on their sides.
Exercise #121

Standard: Data Analysis and Probability

Indicator: DAP #1 (7th Grade)
Read, create and interpret box-and-whisker plots, stem-and-leaf plots, and other types of graphs.

Alexandra’s Favorite Snack Foods

<table>
<thead>
<tr>
<th>Snack Food</th>
<th>Fat Content (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potato Chips</td>
<td>25</td>
</tr>
<tr>
<td>Pretzels</td>
<td>4</td>
</tr>
<tr>
<td>Popcorn</td>
<td>21</td>
</tr>
<tr>
<td>Corn Chips</td>
<td>33</td>
</tr>
<tr>
<td>Nacho Chips</td>
<td>22</td>
</tr>
<tr>
<td>Tortilla Chips</td>
<td>26</td>
</tr>
<tr>
<td>Pistachios</td>
<td>20</td>
</tr>
<tr>
<td>Peanuts</td>
<td>24</td>
</tr>
<tr>
<td>Fried Pork Skins</td>
<td>25</td>
</tr>
<tr>
<td>Cheese Popcorn</td>
<td>31</td>
</tr>
</tbody>
</table>

1. Create a box-and-whisker plot of the data. Label the following points on the plot: median, upper quartile, lower quartile, lower extreme, upper extreme.

Grams of Fat in Snack Foods

0 10 20 30 40

Continue→
2. For this data set, describe what information the median gives. Be specific for this set of data.

3. For this data set, describe what information the lower quartile (also called the 1st quartile) gives. Be specific for this set of data.

4. Does this data set contain any outliers? Explain why the value(s) you choose could be an outlier(s).
Scoring Guide
Point Value: 10 Points (4 + 2 + 2 + 2)

Question 1:
4 Points: Response includes the correctly drawn plot and labels.

**Plot:**
- Lower extreme at 4
- Lower quartile at 21
- Median at 24.5
- Upper quartile at 26
- Upper extreme at 33

**Corresponding Labels:**
- Lower Extreme
- Lower Quartile
- Median
- Upper Quartile
- Upper Extreme

3 Points: Response contains 1 error in either the “Plot” or “Labels” category.

2 Points: Response contains 2 errors in either the “Plot” or “Labels” category OR includes 1 error in each category.

1 Point: Response contains at least 2 errors in one category and 1 error in the other category.

0 Points: Response contains at least 2 errors in each category.

Question 2:
2 Points: Student explains the meaning of the median in this situation. For example, “Half of the snacks have less fat than the median values and half of the snacks have more than the median value.”

1 Point: Student gives a general explanation of the median. For example, “Half of the numbers are below the median and half of the numbers are above it.”

0 Points: Incorrect or no response.

Question 3:
2 Points: Student explains the meaning of the lower quartile in this situation. For example, “25% of the snacks have less fat than the lower quartile value and 75% of the snacks have more fat than the lower quartile value.”

1 Point: Student gives a general explanation of the lower quartile. For example, “The lower quartile separates the bottom 25% from the top 75%.”

0 Points: Incorrect or no response.

Question 4:
2 Points: Student states that 4 (Pretzels) is an outlier. Student explains that it is much lower than any of the other fat values.

1 Point: Student states that 4 (Pretzels) is an outlier, but does not give an appropriate explanation. OR Student gives a correct explanation of an outlier, but does not give the correct outlier in this situation.

0 Points: Incorrect or no response.
Exercise #122

The interior angles of triangle $RST$ are $87^\circ$, $57^\circ$, and $36^\circ$.

Which angle has a measure of $36^\circ$? How do you know?
Scoring Guide

Point Value: 2

2 Points: Response includes the following information:

- Angle RST has a measure of 36°.
- Student explains that the smallest angle of a triangle will be opposite the shortest side.

1 Point: Response contains 1 of the 2 parts of the 2-point response.

0 Points: Response contains 0 of the 2 parts of the 2-point response.
Exercise #123

Standard: Geometry and Spatial Sense

Indicator: Geometry #5 (Grade 7)

Apply properties of congruent or similar triangles to solve problems involving missing lengths and angle measures.

1. What is the measure of \( \angle QPT \)?
   - a. 36°
   - b. 57°
   - c. 87°
   - d. 93°

2. What is the measure of \( \angle QRT \)?
   - a. 87°
   - b. 90°
   - c. 93°
   - d. 123°
Answer Key

Point Value: 2 (1 point each)

1. B. 57°

2. C. 93°
Exercise #124

Standard: Number, Number Sense and Operations

Indicator: NNSO #4 (7th Grade)

Use order of operations and properties to simplify numerical expressions involving integers, fractions, and decimals.

Evaluate the expressions. Show your work.

a. $5.25(5 + 7) - 4.5$

b. $\frac{(15 - 9) \cdot 4}{21 \div 7}$

c. $(20 - 8 \frac{1}{8}) - 3 \cdot \frac{1}{4}$
Scoring Guide

Point Value: 6 Points (2 points each)

a. 58.5 (one point for correct answer; one point for showing work)
b. 8 (one point for correct answer; one point for showing work)
c. $11\frac{1}{8}$ (one point for correct answer; one point for showing work)
Exercise #125

Standard: Number, Number Sense and Operations

Indicator: NNSO #4 (7th Grade)

Use order of operations and properties to simplify numerical expressions involving integers, fractions, and decimals.

Complete each equation using knowledge of properties. Then, label each with the name of the property. Use the following properties: Commutative, Associative, Distributive, Identity of Multiplication, Identity of Addition, Inverse of Addition, Inverse of Multiplication, and Zero Property of Multiplication.

a. \((3+9)y = 3y + _____\) ____________________________

b. \(_____ \cdot 1 = 10\) ____________________________

c. \(7.5 + _____ = 0\) ____________________________

d. \(-7 \cdot _____ = 1\) ____________________________

e. \(-6 \cdot _____ = b \cdot -6\) ____________________________

f. \(6(z+\frac{2}{3}) = 6z + _____\) ____________________________

g. \(c + _____ = c\) ____________________________

h. \(\frac{1}{9} \cdot (9 \cdot 71) = (\frac{1}{9} \cdot 9) \cdot _____\) ____________________________

i. \(-4 \cdot _____ = 0\) ____________________________

j. \(-5 + _____ = 0\) ____________________________
Scoring Guide

Point Value: 10 (1 point each)

a. $(3+9)y = 3y + 9y$  distributive

b. $10 \times 1 = 10$  identity of multiplication

c. $7.5 + (-7.5) = 0$  inverse of addition

d. $-7 \times -1/7 = 1$  inverse of multiplication

e. $-6 \times b = b \times -6$  commutative of multiplication

f. $6(x+\frac{2}{3}) = 6x + (6 \times \frac{2}{3})$  distributive

g. $c + 0 = c$  identity of addition

h. $\frac{1}{9} \cdot (9 \cdot 71) = (\frac{1}{9} \cdot 9) \cdot 71$  associative of multiplication

i. $-4 \times 0 = 0$  zero property of multiplication

j. $-5 + 5 = 0$  inverse of addition
Exercise #126

Standard: Number, Number Sense and Operations
Indicator: NNSO #5 (7th Grade)

Explain the meaning and effect of adding, subtracting, multiplying and dividing integers.

Jasmine and Monique made up a game to help them learn how to add negative and positive integers. The game takes place on a number line.

Rules of the game: Everyone starts at zero. Everyone faces to the right. On your turn you draw two numbers. If you draw a positive number, go that many steps forward. If you draw a negative number, go that many steps backward.

Jasmine went first. She got a 4 and then a -6. So she moved her giraffe, 4 steps forward and then 6 steps back.

On Monique’s turn, she got a -2 and a 5.
In the space below, answer the following questions related to Jasmine and Monique’s new game.

a. What number did Jasmine’s piece end up on?

b. What number did Monique’s piece end up on?

c. Write an equation that represents Jasmine’s movements.

d. Write an equation that represents Monique’s movements.
Scoring Guide

Point Value: 4 Points

4 Points: The response includes the following 4 elements:
- Jasmine’s piece ends on -2;
- Monique’s piece ends on +3;
- Equation for Jasmine’s movements: 4 + -6 = -2
- Equation for Monique’s movements: -2 + 5 = +3

3 Points: The response includes 3 of the elements of a 4-point response.

2 Points: The response includes 2 of the elements of a 4-point response.

1 Point: The response includes 1 of the elements of a 4-point response.

0 Points: The response shows no understanding of the concept.

Teacher Note: This problem simulates how students can use a number line to add integers.

This game can be done in class by using masking tape to tape a large number line on the floor. Have students volunteer to be the walker/jumper. If students follow the rules of the “game”, they will be able to solve problems with integers that involve any of the four operations. Tell students that the signs on the numbers tell them which direction to walk or jump on the number line. For addition and subtraction, students begin by standing on zero and facing the positive direction. They walk forward if the first number of the problem is positive, they walk backward if the first number of the problem is negative. If the problem is an addition problem they continue facing positive and then walk either forward (+) or backward (-) for the second number in the problem. In subtraction problems, students turn around after walking the first number to face in the negative direction to walk the second number. Students should also record this process on a number line on paper.
Exercise #127

Standard: Number, Number Sense and Operations

Indicator: NNSO #5 (7th Grade)
Explain the meaning and effect of adding, subtracting, multiplying and dividing integers.

Frank modified Jasmine and Monique’s game so that they could practice **subtracting** negative and positive integers.

Rules of the game: Everyone starts at zero. Everyone faces to the right. Draw your first number and move your piece the same as you would in Monique and Jasmine’s game. **Then turn your piece to face left** and draw your second number and move your piece. For both numbers, if you draw a positive number, go that many steps forward. If you draw a negative number, go that many steps backward.

Frank went first. He got a 3 and then a 2. So he moved his giraffe, 3 steps forward, turned his piece and moved 2 steps forward.

On Jasmine’s turn, she got a -2 and then a 2.
In the space below, answer the following questions related to Frank’s new version of Jasmine and Monique’s game.

a. What number did Frank's piece end up on?
b. What number did Jasmine’s piece end up on?
c. Write an equation that represents Frank’s movements.
d. Write an equation that represents Jasmine’s movements.
Scoring Guide

Point Value: 4 Points

4 Points: The response includes the following 4 elements:
- Frank’s piece ends on +1;
- Jasmine’s piece ends on -4;
- Equation for Frank’s movements: 3 + 2 = +1
- Equation for Jasmine’s movements: -2 - 2 = -4

3 Points: The response includes 3 of the elements of a 4-point response.

2 Points: The response includes 2 of the elements of a 4-point response.

1 Point: The response includes 1 of the elements of a 4-point response.

0 Points: The response shows no understanding of the concept.

**Teacher Note:** This problem simulates how students can use a number line to subtract integers.

This game can be done in class by using masking tape to tape a large number line on the floor. Have students volunteer to be the walker/jumper. If students follow the rules of the “game”, they will be able to solve problems with integers that involve any of the four operations. Tell students that the signs on the numbers tell them which direction to walk or jump on the number line. For addition and subtraction, students begin by standing on zero and facing the positive direction. They walk forward if the first number of the problem is positive, they walk backward if the first number of the problem is negative. If the problem is an addition problem they continue facing positive and then walk either forward (+) or backward (-) for the second number in the problem. In subtraction problems, students turn around after walking the first number to face in the negative direction to walk the second number. Students should also record this process on a number line on paper.
Exercise #128

Standard: Number, Number Sense and Operations

Indicator: NNSO #5 (7th Grade)
Explain the meaning and effect of adding, subtracting, multiplying and dividing integers.

- Integer counters can help us to solve arithmetic problems. Each white counter has the value of one positive unit, while each gray counter has the value of one negative unit.

For example, \( \begin{array}{c}
\text{white counters} \\
\text{gray counters}
\end{array} \) equals +3, and \( \begin{array}{c}
\text{white counters} \\
\text{gray counters}
\end{array} \) equals -3. When a white counter and a gray counter are put together, they create “zero” pairs (cancel each other) just like -1 and +1.

\[
\begin{array}{c}
\text{white counters} \\
\text{gray counters}
\end{array} = \text{zero} \quad \longleftrightarrow \quad 1 + (-1) = 0
\]

- Jameela used counters to help her solve an addition problem.

\[-7 + 10 = \]

\[
\begin{array}{c}
\text{gray counters} \\
\text{white counters}
\end{array}
\]

She started with -7.

\[
\begin{array}{c}
\text{gray counters} \\
\text{white counters}
\end{array}
\]

She added +10.

\[
\begin{array}{c}
\text{gray counters} \\
\text{white counters}
\end{array}
\]

She canceled out the pairs and ended up with a solution of +3.

- On the next page, use counters to show how you would solve addition problems. Show pictures, explain each step, and give your solution.

Continue on the next page…
1. \( 8 + -3 = \)

2. \( -2 + -6 = \)
Scoring Guide

Total Points: 6 (4 points + 2 Points)

Question 1

2 Points: Response includes the following process.

<table>
<thead>
<tr>
<th>Picture Format</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>Eight white counters represent 8. Add three gray counters to represent addition of -3.</td>
</tr>
<tr>
<td>Step 2</td>
<td>Cancel out any pairs. Five white counters remain.</td>
</tr>
</tbody>
</table>

Solution is 5

1 Point: Response contains ONE error in the steps of the complete response.

0 Points: No response OR response contains TWO or more errors in the steps of the complete response OR response demonstrates no understanding of the concept.

Question 2

2 Points: Response includes the following process.

<table>
<thead>
<tr>
<th>Picture Format</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>Begin with two gray circles to represent -2. Add 6 more gray counters to represent addition of -6.</td>
</tr>
<tr>
<td>Step 2</td>
<td>No pairs cancel. There are 8 gray circles.</td>
</tr>
</tbody>
</table>

Solution is -8

1 Point: Response contains ONE error in the steps of the complete response.

0 Points: No response OR response contains TWO or more errors in the steps of the complete response OR response demonstrates no understanding of the concept.
Exercise #129

Standard: Data Analysis and Probability
Indicator: DAP #7 (7th Grade)
Compute probabilities of compound events.

A bag contains 4 blue and 2 green marbles. Raphael draws one marble and does not replace it. Then he draws another marble and does not replace it.

Show all of the possible outcomes for the two draws.

Are the events independent or dependent events? Explain how you know.

What is the probability that Raphael draws a blue marble first, and a green marble second.
Scoring Guide

Point Value: 4 Points

4 points: The response includes both of the following elements:
- The response correctly shows all possible outcomes (see below);
- The response indicates that the events are dependent because the marbles are not replaced after each draw.
- The response shows that the probability of drawing a blue marble then a green marble is 4/15.
- The response shows how they got the correct answer.

3 points The response includes 3 of the elements of a 4-point response.

2 points The response includes 2 of the elements of a 4-point response.

1 points The response includes 1 of the elements of a 4-point response.

0 points The response includes none of the elements of a 4-point response.

Teacher Notes:

The probability of drawing a blue marble on the first draw is equal to 4/6, the number of blue marbles in the bag, 4, divided by the total number of marbles in the bag, 6. Since marbles are not replaced between draws, the second draw is a dependent event. The second draw is dependent on the outcome of the first draw. The probability of drawing a green marble on the second draw is 2/5, the number of green marbles in the bag, 2, divided by the total number of marbles in the bag, which is now 5. Therefore, the probability of drawing one blue marble then one green marble is 4/6 * 2/5 = 8/30 = 4/15.
Exercise #130

Standard: Data Analysis and Probability

Indicator: DAP #7 (7th Grade)
Compute probabilities of compound events.

Rodrigo rolls a number cube 30 times in an experiment. His actual results are shown in the table below.

<table>
<thead>
<tr>
<th>Number Rolled on Cube</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Times Rolled</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

1. What is the theoretical probability of rolling a 2?

   O A. 1/5  
   O B. 1/30  
   O C. 1/2  
   O D. 1/6

2. What is the experimental probability of rolling a 2?

   O A. 1/5  
   O B. 1/30  
   O C. 1/2  
   O D. 1/6
Answer Key
Point Value: 2 Points (1 point each)

1. D
2. A
### Ohio Grade 8 Mathematics Achievement Test Reference Sheet

Information that may be needed to solve problems on the Mathematics Test:

<table>
<thead>
<tr>
<th>Area Formulas</th>
<th>Volume Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>parallelogram</strong></td>
<td>cone</td>
</tr>
<tr>
<td>$A = bh$</td>
<td>$V = \frac{1}{3}\pi r^2h$</td>
</tr>
<tr>
<td><strong>rectangle</strong></td>
<td>cylinder</td>
</tr>
<tr>
<td>$A = lw$</td>
<td>$V = \pi r^2h$</td>
</tr>
<tr>
<td><strong>trapezoid</strong></td>
<td>pyramid</td>
</tr>
<tr>
<td>$A = \frac{1}{2}h(b_1 + b_2)$</td>
<td>$V = \frac{1}{3}Bh$</td>
</tr>
<tr>
<td><strong>triangle</strong></td>
<td>right prism</td>
</tr>
<tr>
<td>$A = \frac{1}{2}bh$</td>
<td>$V = Bh$</td>
</tr>
</tbody>
</table>

- \(B = \text{area of base}\)

<table>
<thead>
<tr>
<th>Circle Formulas</th>
<th>Sphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C = 2\pi r$</td>
<td>$V = \frac{4}{3}\pi r^3$</td>
</tr>
<tr>
<td>(\pi \approx 3.14)</td>
<td></td>
</tr>
<tr>
<td>(\pi \approx \frac{22}{7})</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Distance Formula</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$</td>
<td></td>
</tr>
</tbody>
</table>