Cleveland Heights-University Heights City School District
2155 Miramar Boulevard
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# MATHEMATICS

Course of Study
Grades K-12

Carlton J. Moody, Superintendent

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# Math Task Force

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| Brinkman | Darwin | | Wiley Middle |
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**DISTRICT PHILOSOPHY**

PHILOSOPHY OF EDUCATION

The educational organization, administration, development, and programs within our school system are to be guided by the following premises:

1.      All children are entitled to public education that will encourage the fullest development of their individual talents.

2.      Learning is essential to individual independence and the general welfare of the changing American society.

3.      Effective learning develops both intellectual and emotional qualities and is for action; its proper evaluation lies in the quality of resultant activity, not mere response.

4.      Organized education, a deployment of human and material resources as means toward learning, implies learning for all (students, teachers, administrators) at different levels of understanding and maturity.

5.      Organized education is at its best when graced with experimental attitudes and dedicated to the proposition that all issues between organization and learning shall be resolved in favor of learning.

6.      All rewards and penalties, adopted in the organization of education, become legitimate only insofar as they assist learning.

7.      For purposes of learning and growth, internal motivations are more valuable than the external, such as grades and competition.

The primary people in our school system are first, the students, and next, the teachers.  Accordingly, they will be given primary consideration in any action taken by the Board of Education.

# Introduction

The Mathematics course of study is based on the Ohio Academic Content Standards.

Ohio's Mathematics academic content standards serve as a basis for what all students should know and be able to do by the time they graduate from high school. These standards, benchmarks and grade-level indicators are intended to provide Ohio educators with a set of common expectations from which to base mathematics curriculum.

These standards:

• Set high expectations and strong support for mathematics achievement by ALL students.

• Represent mathematics knowledge and skills needed to make a successful transition to post-secondary education, workplace and daily life.

• Reflect sound application of research on how students learn mathematics.

• Align with national standards documents and major studies in mathematics.

• Address mathematics content knowledge and mathematical processes, including problem-solving, mathematical reasoning, communication, representation and connections.

• Focus on important mathematics topics that are well-articulated through benchmarks and grade-level indicators.

• Represent rigorous progression across grades and in-depth study within each grade.

• Provide an appropriate balance among conceptual understanding, procedural knowledge and skills, and application and problem-solving.

• Incorporate use of technology by ALL students in learning mathematics.

• Serve as the basis for classroom, district and state assessments.

The mathematics academic content standards prepare all students for success in the workplace and post-secondary education. Competency in mathematics includes understanding of mathematical concepts, facility with mathematical skills, and application of concepts and skills to problem-solving situations. Students are able to communicate mathematical reasoning using mathematical and everyday language.

Whenever possible, students should have opportunities to learn mathematics through real-world contexts, including practical applications, real data, and numbers often associated with situations and problems encountered in the workplace and daily life. All students should be exposed to a mathematics program rich in technology, including calculators, computers, and technology applications.

The six standards that follow represent the mathematics content and processes all students should know and be able to use as they progress through school. These include:

Content Standards:    Number, Number Sense and Operations
                     Measurement
                     Geometry and Spatial Sense
                     Patterns, Functions and Algebra
                     Data Analysis and Probability

Process Standard:    Mathematical Processes

The rigorous, yet realistic standards provide a comprehensive foundation for all students to think and reason mathematically and use mathematics knowledge and skills effectively in post-secondary education, the workplace, and daily life. These standards represent a connected body of mathematical understandings and competencies, rather than a menu of discrete topics from which to choose.

Throughout the five content standards, students will use mathematical processes, including reasoning, communication and representation skills, and appropriate technology within problem-solving situations. Making connections within mathematics and between mathematics and other disciplines is critical for student success in using mathematics effectively in school, work and daily life.

Even though each of these six standards applies to all grades, emphasis will vary both within and between the grade clusters. For example, the emphasis on number, number sense and operations is greatest in elementary grades and by grades 9-12, topics related to number, number sense and operations represent a smaller portion of the curriculum.

This set of six standards does not neatly separate the curriculum into separate or discrete topics. The content described by the standards and benchmarks are interrelated. For example, number pervades all areas of mathematics, and some topics in measurement are closely related to those within geometry. And mathematical processes are used in all content areas and are best learned when carefully integrated with content in instruction and assessment.

Technology, such as calculators and computers, help students learn mathematics and support effective mathematics teaching. Rather than replacing the learning of basic concepts and skills, technology can connect skills and procedures to deeper mathematical understanding. For example, geometry software allows experimentation with families of geometric objects, and graphing utilities facilitate learning about the characteristics of classes of functions.

The following terms and definitions are used in the document:

Standard: An overarching goal or theme in mathematics. The standard statement describes, in broad terms, what students should know and be able to do as a result of the K-12 program.

Benchmark: A specific statement of what a student should know and be able to do at a specific time in his/her schooling. Benchmarks are used to measure a student's progress towards meeting the standard. Benchmarks are defined for grades 2, 4, 7, 10 and 12.

Grade-level Indicator: A specific statement of the knowledge and/or skills that a student demonstrates at each grade level. These indicators serve as checkpoints that monitor progress toward the benchmarks.

# K-12 Mathematics Benchmarks

By the end of the K-2 program:

| Number, Number Sense and Operations | Measurement | Geometry and Spatial Sense |
|---|---|---|
| A. Use place value concepts to represent whole numbers using numerals, words and physical models.<br><br>B. Recognize, classify, compare and order whole numbers.<br><br>C. Represent commonly used fractions using words and physical models.<br><br>D. Determine the value of a collection of coins and dollar bills.<br><br>E. Make change using coins for values up to one dollar.<br><br>F. Count, using numerals and ordinal numbers.<br><br>G. Model, represent and explain addition as combining sets and counting on.<br><br>H Model, represent and explain subtraction as comparison, take-away and part-to-whole.<br><br>I. Model, represent and explain multiplication as repeated addition, rectangular arrays and skip counting.<br><br>J. Model, represent and explain division as sharing equally, repeated subtraction and rectangular arrays. | A. Explain the need for standard units of measure.<br><br>B. Select appropriate units for length, weight, volume (capacity) and time, using:<br>• objects; i.e., non-standard units;<br>• U.S. customary units: inch, foot, yard, ounce, pound, cup, quart, gallon, minute, hour, day, week and year;<br>• metric units: centimeter, meter, gram and liter.<br><br>C. Develop common referents for units of measure for length, weight, volume (capacity) and time to make comparisons and estimates.<br><br>D. Apply measurement techniques to measure length, weight and volume (capacity).<br><br>E. Recognize that using different units of measurement will yield different numbers for the same measurement. | A. Describe and create plane figures: circle, rectangle, square, triangle, hexagon, trapezoid, parallelogram and rhombus, and identify them in the environment.<br><br>B. Describe solid objects: cube, rectangular prism, sphere, cylinder, cone and pyramid, and identify them in the environment.<br><br>C. Sort and compare two-dimensional figures and three-dimensional objects according to their characteristics and properties.<br><br>D. Identify, explain and model (superposition, copying) the concept of shapes being congruent and similar.<br><br>F. Recognize two- and three-dimensional objects from different positions.<br><br>F. Describe location, using comparative (before, after), directional (above, below), and positional (first, last) words.<br><br>G. Identify and draw figures with line symmetry. |

# K-12 Mathematics Benchmarks

By the end of the K-2 program:

| Patterns, Functions and Algebra | Data Analysis and Probability | Mathematical Processes |
|---|---|---|
| A. Sort, classify and order objects by size, number and other properties, and describe the attributes used. | A. Pose questions and gather data about everyday situations and familiar objects. | A. Use a variety of strategies to understand problem situations; e.g., discussing with peers, stating problems in own words, modeling problems with diagrams or physical materials, identifying a pattern. |
| B. Extend sequences of sounds and shapes or simple number patterns, and create and record similar patterns. | B. Sort and classify objects by attributes, and organize data into categories in a simple table or chart. | B. Identify and restate in own words the question or problem and the information needed to solve the problem. |
| C. Create and extend patterns, and describe the rule in words. | C. Represent data using objects, picture graphs and bar graphs. | C. Generate alternative strategies to solve problems. |
| D. Model problem situations, using objects, pictures, numbers and other symbols. | D. Describe the probability of chance events as more, less or equally likely to occur. | D. Evaluate the reasonableness of predictions, estimations and solutions. |
| E. Solve open sentences and explain strategies. | | E. Explain to others how a problem was solved. |
| F. Represent an unknown quantity as a variable using a symbol, such as $\square$, $\Delta$, O | | F. Draw pictures and use physical models to represent problem situations and solutions. |
| G. Describe and compare qualitative and quantitative changes. | | G. Use invented and conventional symbols and common language to describe a problem situation and solution. |

# K-12 Mathematics Benchmarks

By the end of the K-2 program:

| Number, Number Sense and Operations | Measurement | Geometry and Spatial Sense |
|---|---|---|
| K. Demonstrate fluency in addition facts with addends through 9 and corresponding subtractions. | | |
| L  Demonstrate fluency in adding and subtracting multiples of 10, and recognize combinations that make 10. | | |
| M. Add and subtract two-digit numbers with and without regrouping. | | |

# K-12 Mathematics Benchmarks
By the end of the K-2 program:

| Patterns, Functions and Algebra | Data Analysis and Probability | Mathematical Processes |
|---|---|---|
| | | H. Recognize the mathematical meaning of common words and phrases, and relate everyday language to mathematical language and symbols.<br><br>I. Communicate mathematical thinking by using everyday language and appropriate mathematical language. |

# Kindergarten

## Number, Number Sense and Operations Standard

*Number and Number Systems*

1. Compare and order whole numbers up to 10. (Vocabulary: before and after)

2. Explain rules of counting, such as each object should be counted once and that order does not change the number.

3. Count to twenty; e.g., in play situations or while reading number books. Introduce counting to 100

4. Determine "how many" in sets (groups) of 10 or fewer objects.

5. Read and write numerals for single-digit numbers (0 to 20).

6. Construct multiple sets of objects each containing the same number of objects.

7. Compare the number of objects in two or more sets when one set has one or two more, or one or two fewer objects. Vocabulary: greater than, less than

*8.* Represent using manipulatives and use whole numbers in flexible ways, including relating, composing and decomposing numbers; e.g., 5 marbles can be 2 red and 3 green or 1 red and 4 green.

*Meaning of Operations*

9. Identify and state the value of a penny, nickel and dime, and quarter

10. Model and represent addition as combining sets and counting on, and subtraction as take-away and comparison. For example:
    a. Combine and separate small sets of objects in contextual situations; e.g., add or subtract one, two, or another small amount.
    b. Count on (forward) and count back (backward) on a number line between 0 and 10.

11. Demonstrate joining multiple groups of objects, each containing the same number of objects; e.g., combining 3 bags of candy, each containing 2 pieces.

*Computation and Estimation*

12. Partition or share a small set of objects into groups of equal size; e.g., sharing 6 stickers equally among 3 children.

13. Recognize the number or quantity in sets up to 6 without counting; with instant recognition e.g., recognize without counting the dot arrangement on a domino.

## Measurement Standard

1. Identify units of time (day, week, month, year) and compare calendar
   elements; e.g., weeks are longer than days.

2. Compare and order objects of different lengths, areas, weights and capacities;
   and use relative terms, such as longer, shorter, bigger, smaller, heavier,
   lighter, more and less.

3. Measure length and volume (capacity) using uniform objects in the
   environment. For example, find:

   a. how many paper clips long is a pencil;
   b. how many small containers it takes to fill one big container using sand,
      rice, beans.

4. Order events based on time. For example:

   a. activities that take a long or short time;
   b. review what we do first, next, last;
   c. recall what we did or plan to do yesterday, today, tomorrow.

5. Introduce telling time to the hour on digital and analog timepieces


## Geometry and Spatial Sense Standard

1. Identify and sort two-dimensional shapes and three-dimensional objects. For
   example:

   a. Identify and describe two-dimensional figures and three-dimensional
      objects from the environment using the child's own vocabulary.

   b. Sort shapes and objects into groups based on student-defined categories.

   c. Select all shapes or objects of one type from a group.

   d. Build two-dimensional figures using paper shapes, tangrams, or
      patternblocks; build simple three-dimensional objects using blocks, such
      as geoblocks.

2. Name and demonstrate the relative position of objects as follows:

   a. place objects over, under, inside, outside, on, beside, between, above,
      below, on top of, upside-down, behind, in back of, in front of;
   b. describe placement of objects with terms, such as on, inside, outside,
      above, below, over, under, beside, between, in front of, behind.

**Patterns, Functions and Algebra Standards**

*Use Patterns, Relations and Functions*

1. Sort, classify and order objects by size, number and other properties. For example:

   a. Identify how objects are alike and different.
   b. Order three events or objects according to a given attribute, such as time or size.
   c. Recognize and explain how objects can be classified in more than one way.
   d. Identify what attribute was used to sort groups of objects that have already been sorted.

2. Identify, create, extend and copy sequences of sounds (such as musical notes), shapes (such as buttons, leaves or blocks), motions (such as hops or skips), and numbers from 1 to 10.

3. Describe orally the pattern of a given sequence.

4. Model a problem situation using physical materials.

*Use Algebraic Representations*

**Data Analysis and Probability Standard**

*Data Collection*

1. Gather and sort data in response to questions posed by teacher and students; e.g., how many sisters and brothers, what color shoes.

2. Arrange objects in a floor or table graph according to attributes, such as use, size, color or shape.

*Statistical Methods*

3. Select the category or categories that have the most or fewest objects in a floor or table graph.

# Grade One
## Number, Number Sense and Operations Standard

*Number and Number Systems*

1.  Use ordinal numbers to order objects; e.g., first, second, third. (Vocabulary before, after, and between*)*

2.  Recognize and generate equivalent forms for the same number using physical models, words and number expressions; e.g., concept of ten is described by "10 blocks," full tens frame, numeral 10, 5 + 5, 15 - 5, one less than 11, my brother's age.

3.  Read and write the numerals for numbers to 100.

4.  Count forward to 100, count backwards from 100, and count or backward starting at any number between 1 and 100.

5.  Use place value concepts to represent whole numbers using numerals, words, expanded notation and physical models with ones and tens. For example:
    a.  Develop a system to group and count by twos, fives and tens.
    b.  Identify patterns and groupings in a 100's chart and relate to place value concepts.
    c.  Recognize the first digit of a two-digit number as the most important to indicate size of a number and the nearness to 10 or 100.

6.  Identify and state the value of a penny, nickel, dime, quarter and dollar.

7.  Determine the value of a small collection of coins (with a total value up to one dollar) using 1 or 2 different type coins, including pennies, nickels, dimes and quarters.

8.  Show different combinations of coins that have the same value u*p* to $1

9.  Represent commonly used fractions using words and physical models for halves, thirds and fourths, recognizing fractions are represented by equal size parts of a whole and of a set of objects.

10. Model, represent and explain addition as combining sets (part + part = whole) and counting on. For example:
    a.  Model and explain addition using physical materials in contextual situations.
    b.  Draw pictures to model addition.
    c.  Write number sentences to represent addition.
    d.  Explain that adding two whole numbers yields a larger whole number.

*Meaning of Operations*

11. Model, represent and explain subtraction as take-away and comparison. For example:
    a.  Model and explain subtraction using physical materials in contextual situations.
    b.  Draw pictures to model subtraction.

c. Write number sentences to represent subtraction.
d. Explain that subtraction of whole numbers yields an answer smaller than the original number.

**12.** Use conventional symbols to represent the operations of addition and subtraction, written horizontally and vertically

13. Model and represent multiplication as repeated addition and rectangular arrays in contextual situations; e.g., four people will be at my party and if I want to give 3 balloons to each person, how many balloons will I need to buy?

14. Model and represent division as sharing equally in contextual situations; e.g., sharing cookies.

**15.** Demonstrate that equal means "the same as" using visual representations.

*Computation and Estimation*

16 Recognize, classify, compare and order whole numbers up to 100 (Vocabulary: greater than, less than < >)

17. Develop strategies for basic addition facts, such as:
  a. counting all;
  b. counting on;
  c. one more, two more;
  d. doubles;
  e. doubles plus or minus one;
  f. make ten;
  g. models of 10;
  h. identity property (adding zero).

18. Develop strategies for basic subtraction facts, such as:
  a. relating to addition (for example, think of 7 - 3 = ? as "3 plus ? equals 7");
  b. one less, two less;
  c. all but one (for example, 8 - 7, 5 - 4);
  d. models of 10;
  e. missing addends.

## Measurement Standard

*Measurement Units*

1. Recognize and explain the need for fixed units and tools for measuring length and weight; e.g., rulers and balance scales.

2. Tell time to the hour and half hour on digital and analog (dial) timepieces.

3. Order a sequence of events with respect to time; e.g., summer, fall, winter and spring; morning, afternoon and night.

*Use Measurement Techniques and Tools*

4. Estimate and measure weight using non-standard units; e.g., blocks of uniform size.

5. Estimate and measure lengths using non-standard units such as paperclips, or hands and standard units such as centimeters, inches and feet.

## Geometry and Spatial Sense Standard

*Characteristics and Properties*

1. Identify, compare and sort two-dimensional shapes; i.e., square, circle, ellipse, triangle, rectangle, rhombus, trapezoid, parallelogram, pentagon and hexagon, using manipulatives such as pattern blocks.

   a. Recognize and identify triangles and rhombuses independent of position, shape or size;
   b. Describe two-dimensional shapes using attributes such as number of sides and number of vertices (corners or angles).

2. Create new shapes by combining or cutting apart existing shapes.

3. Identify the shapes of the faces of three-dimensional objects.

*Spatial Relationships*

4. Extend the use of location words to include distance (near, far, close to) and directional words (left, right).

   5  Copy figures and draw simple two-dimensional shapes from memory.

6. Explore line symmetry with concrete materials.

## Patterns, Functions and Algebra Standard

*Use Patterns, Relations and Functions*

1. Sort, classify and order objects by two or more attributes, such as color and shape, and explain how objects were sorted.

2. Extend sequences of sounds, shapes or simple number patterns, and create and record similar patterns. For example:
   a. Analyze and describe patterns with multiple attributes using numbers and shapes; e.g., AA, B, aa, b, AA, B, aa, b,…
   b. Continue repeating and growing patterns with materials, pictures and geometric items; e.g., XO, XOO, XOOO, XOOOO.

3. Describe orally the basic unit or general plan of a repeating or growing pattern.

*Use Algebraic Representations*

4  Solve open sentences by representing an expression in more than one way

using the commutative property; e.g., 4 + 5 = 5 + 4 or the number of blue balls plus red balls is the same as the number of red balls plus blue balls (R + B = B + R).

5.  Describe orally and model a problem situation using words, objects or number phrase or sentence.

## Data Analysis and Probability Standard

*Data Collection*

1.  Identify multiple categories for sorting data.

2.  Collect and organize data into charts using tally marks.

3.  Display data in picture graphs with units of 1 and bar graphs with intervals of 1.

4.  Read and interpret charts, picture graphs and bar graphs as sources of information to identify main ideas, draw conclusions, and make predictions.

*Statistical Methods*

5. Construct a question that can be answered by using information from a graph.

6.  Arrange five objects by an attribute, such as size or weight, and identify the ordinal position of each object.

7.  Answer questions about the number of objects represented in a picture graph, bar graph or table graph; e.g., category with most, how many more in a category compared to another, how many altogether in two categories.

*Probability*

8.  Describe the likelihood of simple events as possible/impossible and more likely/less likely; e.g., when using spinners or number cubes in classroom activities.

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# Grade Two
## Number, Number Sense and Operations Standard

*Number and Number Systems*

1. Use place value concepts to represent, compare and order whole numbers using physical models, numerals and words, with ones, tens and hundreds. For example:

   a. Recognize 10 can mean "10 ones" or a single entity (1 ten) through physical models and trading games.
   b. Read and write 3-digit numerals (e.g., 243 as two hundred forty three, 24 tens and 3 ones, or 2 hundreds and 43 ones, etc.) and construct models to represent each.

2. Recognize and classify numbers as even or odd.

3. Count money and make change using coins and a dollar bill.

4. Represent and write the value of money using the ¢ sign and in decimal form when using the $ sign.

5. Represent fractions (halves, thirds, fourths, sixths and eighths), using physical models, words, and numerals. For example:

   a. Recognize that a fractional part can mean different amounts depending on the original quantity.
   b. Recognize that a fractional part of a rectangle does not have to be shaded with contiguous parts.
   c. Identify and illustrate parts of a whole and parts of sets of objects.
   d. Compare and order physical models of halves, thirds and fourths in relation to 0 and 1.

*Meaning of Operations*

6. Model, represent and explain subtraction as comparison, take-away and part-to-whole; e.g., solve missing addend problems by counting up or subtracting, such as "I had six baseball cards, my sister gave me more, and I now have ten. How many did she give me?" can be represented as 6 + ? = 10 or 10 - 6 = ?.

7. Model, represent and explain multiplication as repeated addition, rectangular arrays and skip counting.

8. Model, represent and explain division as sharing equally and repeated subtraction.

9. Model and use the commutative property for addition.

*Computation and Estimation*

10. Demonstrate fluency in addition facts with addends through 9 and corresponding subtractions; e.g., 9 + 9 = 18, 18 – 9 = 9.

11. Add and subtract multiples of 10.

12. Demonstrate multiple strategies for adding and subtracting 2- or 3-digit whole numbers, such as:

    a. compatible numbers;
    b. compensatory numbers;
    c. informal use of commutative and associative properties of addition.

13. Estimate the results of whole number addition and subtraction problems using front-end estimation, and judge the reasonableness of the answers.

## Measurement Standard

*Measurement Units*

1. Identify and select appropriate units of measure for:

    a. length – centimeters, meters, inches, feet or yards;
    b. volume (capacity) – liters, cups, pints or quarts;
    c. weight – grams, ounces or pounds;
    d. time – hours, half-hours, quarter-hours or minutes and time designations, a.m. or p.m.

2. Establish personal or common referents for units of measure to make estimates and comparisons; e.g., the width of a finger is a centimeter, a large bottle of soda pop is 2 liters, a small paper clip weighs about one gram.

3. Describe and compare the relationships among units of measure, such as centimeters and meters; inches, feet and yards; cups, pints and quarts; ounces and pounds; and hours, half-hours, and quarter-hours; e.g., how many inches in a foot?

4. Tell time to the nearest minute interval on digital and to the nearest 5 minute interval on analog (dial) timepieces.

*Use Measurement Techniques and Tools*

5. Estimate and measure the length and weight of common objects, using metric and U.S. customary units, accurate to the nearest unit.

6. Select and use appropriate measurement tools; e.g., a ruler to draw a segment 3 inches long, a measuring cup to place 2 cups of rice in a bowl, a scale to weigh 50 grams of candy.

7. Make and test predictions about measurements, using different units to measure the same length or volume.

## Geometry and Spatial Sense Standard

*Characteristics and Properties*

1. Identify, describe, compare and sort three-dimensional objects (i.e., cubes, spheres, prisms, cones, cylinders and pyramids) according to the shape of the faces or the number of faces, edges or vertices.

2. Predict what new shapes will be formed by combining or cutting apart existing shapes.

3. Recognize two-dimensional shapes and three-dimensional objects from different positions.

*Spatial Relationships*

4. Identify and determine whether two-dimensional shapes are congruent (same shape and size) or similar (same shape different size) by copying or using superposition (lay one thing on top of another).

*Transformations and Symmetry*

5. Create and identify two-dimensional figures with line symmetry; e.g., what letter shapes, logos, polygons are symmetrical?

## Patterns, Functions and Algebra Standard

*Use Patterns, Relations and Functions*

1. Extend simple number patterns (both repeating and growing patterns), and create similar patterns using different objects, such as using physical materials or shapes to represent numerical patterns.

2. Use patterns to make generalizations and predictions; e.g., determine a missing element in a pattern.

3. Create new patterns with consistent rules or plans, and describe the rule or general plan of existing patterns.

*Use Algebraic Representations*

4. Use objects, pictures, numbers and other symbols to represent a problem situation.

5. Understand equivalence and extend the concept to situations involving symbols; e.g., $4 + 5 = 9$ and $9 = 4 + 5$, and $4 + 5 = 3 + 6 = \Delta + \square \ldots$

6. Use symbols to represent unknown quantities and identify values for symbols in an expression or equation using addition and subtraction; e.g., $\square + O = 10$, $\Delta - 2 = 4$.

*Analyze Change*

7. Describe qualitative and quantitative changes, especially those involving addition and subtraction; e.g., a student growing taller versus a student growing two inches in one year.

# Data Analysis and Probability Standard

*Data Collection*

1. Pose questions, use observations, interviews and surveys to collect data, and organize data in charts, picture graphs and bar graphs.

2. Read, interpret and make comparisons and predictions from data represented in charts, line plots, picture graphs and bar graphs.

3. Read and construct simple timelines to sequence events.

*Statistical Methods*

4. Write a few sentences to describe and compare categories of data represented in a chart or graph, and make statements about the data as a whole.

5. Identify untrue or inappropriate statements about a given set of data.

6. Recognize that data may vary from one population to another; e.g., favorite TV shows of students and of parents.

*Probability*

7. List some of the possible outcomes of a simple experiment, and predict whether given outcomes are more, less or equally likely to occur.

8. Use physical models and pictures to represent possible arrangements of 2 or 3 objects.

# K-12 Mathematics Benchmarks
By the end of the 3-4 program:

| Number, Number Sense and Operations | Measurement | Geometry and Spatial Sense |
|---|---|---|
| A. Use place value structure of the base-ten number system to read, write, represent and compare whole numbers and decimals.<br><br>B. Recognize and generate equivalent representations for whole numbers, fractions and decimals.<br><br>C. Represent commonly used fractions and mixed numbers using words and physical models.<br><br>D. Use models, points of reference and equivalent forms of commonly used fractions to judge the size of fractions and to compare, describe and order them.<br><br>E. Recognize and classify numbers as prime or composite and list factors.<br><br>F. Count money and make change using both coins and paper bills.<br><br>G. Model and use commutative and associative properties for addition and multiplication. | A. Select appropriate units for perimeter, area, weight, volume (capacity), time and temperature, using:<br>• objects of uniform size;<br>• U.S. customary units; e.g., mile, square inch, cubic inch, second, degree Fahrenheit, and other units as appropriate;<br>• metric units; e.g., millimeter, kilometer, square centimeter, kilogram, cubic centimeter, degree Celsius, and other units as appropriate.<br><br>B. Know that the number of units is inversely related to the size of the unit for any item being measured.<br><br>C. Develop common referents for units of measure for length, weight, volume (capacity) and time to make comparisons and estimates.<br><br>D. Identify appropriate tools and apply counting techniques for measuring side lengths, perimeter and area of squares, rectangles, and simple irregular two-dimensional shapes, volume of rectangular prisms, and time and temperature. | A. Provide rationale for groupings and comparisons of two-dimensional figures and three-dimensional objects.<br><br>B. Describe and identify points, lines and planes in the environment.<br><br>C. Describe and identify intersecting, parallel and perpendicular lines or segments in the environment.<br><br>D. Identify and draw right, obtuse, acute and straight angles.<br><br>E. Use attributes to describe, classify and sketch plane figures and build solid objects.<br><br>F. Develop definitions of classes of shapes.<br><br>G. Find and name locations in coordinate systems.<br><br>H. Identify and describe line and rotational symmetry in two-dimensional shapes and designs. |

# K-12 Mathematics Benchmarks
By the end of the 3-4 program:

| Patterns, Functions and Algebra | Data Analysis and Probability | Mathematical Processes |
|---|---|---|
| A. Analyze and extend patterns, and describe the rule in words. | A. Gather and organize data from surveys and classroom experiments, including data collected over a period of time. | A. Apply and justify the use of a variety of problem-solving strategies; e.g., make an organized list, guess and check. |
| B. Use patterns to make predictions, identify relationships, and solve problems. | B. Read and interpret tables, charts, graphs (bar, picture, line, line plot), and timelines as sources of information, identify main idea, draw conclusions, and make predictions. | B. Use an organized approach and appropriate strategies to solve multi-step problems. |
| C. Write and solve open sentences and explain strategies. | | C. Interpret results in the context of the problem being solved; e.g., the solution must be a whole number of buses when determining the number of buses necessary to transport students. |
| D. Represent an unknown quantity as a variable using a symbol, including letters. | C. Construct charts, tables and graphs to represent data, including picture graphs, bar graphs, line graphs, line plots and Venn diagrams. | |
| E. Use variables to create and solve equations representing problem situations. | | D. Use mathematical strategies to solve problems that relate to other curriculum areas and the real world; e.g., use a timeline to sequence events; use symmetry in artwork. |
| F. Construct and use a table of values to solve problems associated with mathematical relationships. | D. Read, interpret and construct graphs in which icons represent more than a single unit or intervals greater than one; e.g., each $\overline{\text{🚲}}$ = 10 bicycles or the intervals on an axis are multiples of 10. | E. Link concepts to procedures and to symbolic notation; e.g., model 3 x 4 with a geometric array, represent one-third by dividing an object into three equal parts. |
| G. Describe how a change in one variable affects the value of a related variable. | | |
| | E. Describe data using mode, median and range. | |
| | F. Conduct a simple probability experiment and draw conclusions about the likelihood of possible outcomes. | F. Recognize relationships among different topics within mathematics; e.g., the length of an object can be represented by a number. |

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# K-12 Mathematics Benchmarks
By the end of the 3-4 program:

| Number, Number Sense and Operations | Measurement | Geometry and Spatial Sense |
|---|---|---|
| H. Use relationships between operations, such as subtraction as the inverse of addition and division as the inverse of multiplication.<br><br>I. Demonstrate fluency in multiplication facts with factors through 10 and corresponding divisions.<br><br>J. Estimate the results of whole number computations using a variety of strategies, and judge the reasonableness.<br><br>K. Analyze and solve multi-step problems involving addition, subtraction, multiplication and division of whole numbers.<br><br>L. Use a variety of methods and appropriate tools (mental math, paper and pencil, calculators) for computing with whole numbers.<br><br>M. Add and subtract commonly used fractions with like denominators and decimals, using models and paper and pencil. | E. Tell time to the nearest minute. | I. Describe, identify and model reflections, rotations and translations, using physical materials.<br><br>J. Describe a motion or series of transformations that show two shapes are congruent. |

# K-12 Mathematics Benchmarks
By the end of the 3-4 program:

| Patterns, Functions and Algebra | Data Analysis And Probability | Mathematical Processes |
| --- | --- | --- |
| | G. Identify and represent possible outcomes, such as arrangements of a set of up to four members and possible combinations from several sets, each containing 2 or 3 members.<br><br>H. Use the set of possible outcomes to describe and predict events. | G. Use reasoning skills to determine and explain the reasonableness of a solution with respect to the problem situation.<br><br>H. Recognize basic valid and invalid arguments, and use examples and counter examples, models, number relationships, and logic to support or refute.<br><br>I. Represent problem situations in a variety of forms (physical model, diagram, in words or symbols), and recognize when some ways of representing a problem may be more helpful than others.<br><br>J. Read, interpret, discuss and write about mathematical ideas and concepts using both everyday and mathematical language.<br><br>K. Use mathematical language to explain and justify mathematical ideas, strategies and solutions. |

# Grade Three
## Number, Number Sense and Operations Standard

*Number and
Number Systems*

1. Identify and generate equivalent forms of whole numbers;
   e.g., 36, 30 + 6, 9 x 4, 46 - 10, number of inches in a yard.

2. Use place value concepts to represent whole numbers and decimals using numerals, words, expanded notation and physical models. For example:

   a. Recognize 100 means "10 tens" as well as a single entity (1 hundred) through physical models and trading games.
   b. Describe the multiplicative nature of the number system; e.g., the structure of 3,205 as 3 x 1000 plus 2 x 100 plus 5 x 1.
   c. Model the size of 1000 in multiple ways; e.g., packaging 1000 objects into 10 boxes of 100, modeling a meter with centimeter and decimeter strips, or gathering 1000 pop-can tabs.
   d. Demonstrate using physical models the concept of tenths and hundredths such as, base ten blocks, decimal squares or money.

3. Use mathematical language and symbols to compare and order; e.g., less than, greater than, at most, at least, $<$, $>$, $=$, $\leq$, $\geq$.

4. Round whole numbers to the nearest ten or nearest hundred.

5. Count money and make change using coins and paper bills to <u>ten</u> dollars.

6. Represent fractions and mixed numbers using words, numerals and physical models.

7. Compare and order commonly used fractions and mixed numbers using number lines, models (such as fraction circles or bars), points of reference (such as more or less than ½ ), and equivalent forms using physical or visual models.

8. Recognize and use decimal and fraction concepts and notations as related ways of representing parts of a whole or a set; e.g., 3 of 10 marbles are red can also be described as $\frac{3}{10}$ and 3 tenths are red.

*Meaning of
Operations*

9. Model, represent and explain multiplication; e.g., repeated addition, skip counting, rectangular arrays and area model. For example:

   a. Use conventional mathematical symbols to write equations for word problems involving multiplication.
   b. Understand that, unlike addition and subtraction, the factors in multiplication and division may have different units; e.g., 3 boxes of 5 cookies each.

10. Model, represent and explain division; e.g., sharing equally, repeated subtraction, rectangular arrays and area model. For example:

    a.  Translate contextual situations involving division into conventional mathematical symbols.

    b.  Explain how a remainder may impact an answer in a real-world situation; e.g., 14 cookies being shared by 4 children.

11. Explain and use relationships between operations, such as:

    a.  relate addition and subtraction as inverse operations;
    b.  relate multiplication and division as inverse operations;
    c.  relate addition to multiplication (repeated addition);
    d.  relate subtraction to division (repeated subtraction).

*Computation and Estimation*

12. Model and use the commutative and associative properties for addition and multiplication.

13. Add and subtract whole numbers with and without regrouping.

14. Demonstrate fluency in multiplication facts through 10 and corresponding division facts.

15. Multiply and divide 2- and 3-digit numbers by a single-digit number, without remainders for division.

16. Evaluate the reasonableness of computations based upon operations and the numbers involved; e.g., considering relative size, place value and estimates.

17. Estimate the results of whole number addition and subtraction problems using front-end estimation, and judge the reasonableness of the answers

See Definition:  Front-end estimation
Using the leading, or left most, digits to make an estimate quickly and easily. After making an initial estimate using front-end digits, an adjustment can be made to refine the estimate; e.g. Using front-end estimation to estimate the sum of 594, 32, and 221, an initial estimate would be $5 + 0 + 2$ hundreds or 700.  An adjustment can be made by grouping the tens and ones (about $100 + 50$ or 150 more) and adding to get an adjusted estimate of 850.

## Measurement Standard

*Measurement Units*

1. Identify and select appropriate units for measuring:

   a. length – miles, kilometers and other units of measure as appropriate;
   b. volume (capacity) – gallons;
   c. weight – ounces, pounds, grams, or kilograms;
   d. temperature – degrees (Fahrenheit or Celsius).

2. Establish personal or common referents to include additional units; e.g., a gallon container of milk; a postage stamp is about a square inch.

3. Tell time to the nearest minute and find elapsed time using a calendar or a clock.

4. Read thermometers in both Fahrenheit and Celsius scales.

*Use Measurement Techniques and Tools*

5. Estimate and measure length, weight and volume (capacity), using metric and U.S. customary units, accurate to the nearest $\frac{1}{2}$ or $\frac{1}{4}$ unit as appropriate.

6. Use appropriate measurement tools and techniques to construct a figure or approximate an amount of specified length, weight or volume (capacity); e.g., construct a rectangle with length $2\frac{1}{2}$ inches and width 3 inches, fill a measuring cup to the $\frac{3}{4}$ cup mark.

7. Make estimates for perimeter, area and volume using links, tiles, cubes and other models.

## Geometry and Spatial Sense Standard

*Characteristics and Properties*

1. Analyze and describe properties of two-dimensional shapes and three-dimensional objects using terms such as vertex, edge, angle, side and face.

2. Identify and describe the relative size of angles with respect to right angles as follows:

   a. Use physical models, like straws, to make different sized angles by opening and closing the sides, not by changing the side lengths.
   b. Identify, classify and draw right, acute, obtuse and straight angles.

*Spatial Relationships*

3. Find and name locations on a labeled grid or coordinate system; e.g., a map or graph.

*Transformations and Symmetry*

4. Draw lines of symmetry to verify symmetrical two-dimensional shapes.

*Visualization and Geometric Models*   5. Build a three-dimensional model of an object composed of cubes; e.g., construct a model based on an illustration or actual object.

6. Describe points, lines and planes, and identify models in the environment

## Patterns, Functions and Algebra Standard

*Use Patterns, Relations and Functions*   1. Extend multiplicative and growing patterns, and describe the pattern or rule in words.

2. Analyze and replicate arithmetic sequences with and without a calculator.

3. Use patterns to make predictions, identify relationships, and solve problems.

*Use Algebraic Representations*   4. Model problem situations using objects, pictures, tables, numbers, letters and other symbols.

5. Write, solve and explain simple mathematical statements, such as $7 + \Box > 8$ or $\Delta + 8 = 10$, $\Delta \times 2 = 10$, $12 \Box \Box = 3$.

6. Express mathematical relationships as equations and inequalities.

*Analyze Change*   7. Create tables to record, organize and analyze data to discover patterns and rules.

*8.* Identify and describe quantitative changes, especially those involving addition and subtraction; e.g., the height of water in a glass becoming 1 centimeter lower each week due to evaporation.

## Data Analysis and Probability Standard

*Data Collection*   1.   Collect and organize data from an experiment, such as recording and classifying observations or measurements, in response to a question posed.

2. Draw and interpret picture graphs in which a symbol or picture represents more than one object.

3. Read, interpret and construct bar graphs with intervals greater than one.

4. Support a conclusion or prediction orally and in writing, using information in a table or graph.

5. Match a set of data with a graphical representation of the data.

6. Translate information freely among charts, tables, line plots, picture graphs and bar graphs; e.g., create a bar graph from the information in a chart.

*Statistical
Methods*

7. Analyze and interpret information represented on a timeline.

*Probability*

8. Identify the mode of a data set and describe the information it gives about a data set.

9. Conduct a simple experiment or simulation of a simple event, record the results in a chart, table or graph, and use the results to draw conclusions about the likelihood of possible outcomes.

10. Use physical models, pictures, diagrams and lists to solve problems involving possible arrangements or combinations of two to four objects.

# Grade Four
## Number, Number Sense and Operations Standard

*Number and Number Systems*

1. Identify and generate equivalent forms of fractions and decimals. For example:

   a. Connect physical, verbal and symbolic representations of fractions, decimals and whole numbers; e.g., $\frac{1}{2}$, $\frac{5}{10}$, "five tenths," 0.5, shaded rectangles with half, and five tenths.
   b. Understand and explain that ten tenths is the same as one whole in both fraction and decimal form. e.g. $3/3 = 1$  $4/4 = 1$

2. Use place value structure of the base-ten number system to read, write, represent and compare whole numbers through millions and decimals through thousandths.
   a. Recognize 1000 means "10 hundreds" as well as a single entity (1 thousand) through physical models and trading games.
   b. Describe the multiplicative nature of the number system; e.g., 863,205 equals $8 \times 100,000 + 6 \times 10,000 + 3 \times 1,000 + 2 \times 100 + 5 \times 1$ (expanded notation).
   c. Demonstrate using physical models the concept of tenths and hundredths and thousandths, such as base ten blocks, decimal squares.

*Meaning of Operations*

3. Round whole numbers to a given place value.

4. Identify and represent factors and multiples of whole numbers through 100, and classify numbers as prime or composite.

5. Use number line models and points of reference to compare commonly used fractions.

*Computation and Estimation*

6. Use associative and distributive properties to simplify and perform computations; e.g., use left to right multiplication and the distributive property to find an exact answer without paper and pencil, such as $5 \times 47 = 5 \times 40 + 5 \times 7 = 200 + 35 = 235$.

7. Recognize that division may be used to solve different types of problem situations and interpret the meaning of remainders; e.g., situations involving measurement, money.

8. Solve problems involving counting money and making change, using both coins and paper bills.

9. Estimate the results of computations involving whole numbers, fractions and decimals, using a variety of strategies.

10. Use physical models, visual representations, and paper and pencil to add and subtract decimals and commonly used fractions with like denominators.

11. Develop and explain strategies for performing computations mentally.

12. Analyze and solve multi-step problems involving addition, subtraction, multiplication and division using an organized approach, and verify and interpret results with respect to the original problem.

13. Use a variety of methods and appropriate tools for computing with whole numbers; e.g., mental math, paper and pencil, and calculator.

14. Demonstrate fluency in adding and subtracting whole numbers and in multiplying and dividing whole numbers by 1- and 2-digit numbers and multiples of ten.

## Measurement Standard

*Measurement Units*

1. Relate the number of units to the size of the units used to measure an object; e.g., compare the number of cups to fill a pitcher to the number of quarts to fill the same pitcher.

2. Demonstrate and describe perimeter as surrounding and area as covering a two-dimensional shape, and volume as filling a three-dimensional object.

3. Identify and select appropriate units to measure:

   a. perimeter – string or links (inches or centimeters).
   b. area – tiles (square inches or square centimeters).
   c. volume – cubes (cubic inches or cubic centimeters).

*Use Measurement Techniques and Tools*

4. Develop and use strategies to find perimeter using string or links, area using tiles or a grid, and volume using cubes; e.g., count squares to find area of regular or irregular shapes on a grid, layer cubes in a box to find its volume.

5. Make simple unit conversions within a measurement system; e.g., inches to feet, kilograms to grams, quarts to gallons.

6. Write, solve and verify solutions to multi-step problems involving measurement.

7. Find elapsed time using a calendar or a clock.

# Geometry and Spatial Sense Standard

*Characteristics and Properties*

1. Identify, describe and model intersecting, parallel and perpendicular lines and line segments; e.g., use straws or other material to model lines.

2. Describe, classify, compare and model two- and three-dimensional objects using their attributes.

3. Identify similarities and differences of quadrilaterals; e.g., squares, rectangles, parallelograms and trapezoids.

4. Identify and define triangles based on angle measures (equiangular, right, acute and obtuse triangles) and side lengths (isosceles, equilateral and scalene triangles).

5. Describe points, lines and planes, and identify models in the environment.

*Spatial Relationships*

6. Specify locations and plot ordered pairs on a coordinate plane, using first quadrant points.

7. Identify, describe and use reflections (flips), rotations (turns), and translations (slides) in solving geometric problems; e.g., use transformations to determine if 2 shapes are congruent.

*Transformations and Symmetry*

9. Use geometric models to solve problems in other areas of mathematics, such as number (multiplication/division) and measurement (area, perimeter, border).

*Visualization and Geometric Models*

# Patterns, Functions and Algebra Standard

*Use Patterns, Relations and Functions*

1. Use models and words to describe, extend and make generalizations of patterns and relationships occurring in computation, numerical patterns, geometry, graphs and other applications.

2. Represent and analyze patterns and functions using words, tables and graphs.

*Use Algebraic Representations*

3. Construct a table of values to solve problems associated with a mathematical relationship.

4. Use rules and variables to describe patterns and other relationships.

5. Represent mathematical relationships with equations or inequalities.

*Analyze Change*

6. Describe how a change in one variable affects the value of a related variable; e.g., as one increases the other increases or as one increases the other decreases.

## Data Analysis and Probability Standard

1. Create a plan for collecting data for a specific purpose.

2. Represent and interpret data using tables, bar graphs, pictographs, line plots and line graphs.

3. Interpret and construct Venn diagrams to sort and describe data.

4. Compare different representations of the same data to evaluate how well each representation shows important aspects of the data, and identify appropriate ways to display the data.

5. Propose and explain interpretations and predictions based on data displayed in tables, charts and graphs.

6. Describe the characteristics of a set of data based on a graphical representation, such as range of the data, clumps of data, and holes in the data.

7. Identify the median of a set of data and describe what it indicates about the data.

8. Use range, median and mode to make comparisons among related sets of data.

9. Conduct simple probability experiments and draw conclusions from the results; e.g., rolling number cubes or drawing marbles from a bag.

10. Represent the likelihood of possible outcomes for chance situations; e.g., probability of selecting a red marble from a bag containing 3 red and 5 white marbles.

11. Relate the concepts of impossible and certain-to-happen events to the numerical values of 0 (impossible) and 1 (certain).

12. Place events in order of likelihood and use a diagram or appropriate language to compare the chance of each event occurring; e.g., impossible, unlikely, equal, likely, certain.

13. List and count all possible combinations using one member from each of several sets, each containing 2 or 3 members; e.g., the number of possible outfits from 3 shirts, 2 shorts and 2 pairs of shoes.

# K-12 Mathematics Benchmarks

By the end of the 5-7 program:

| Number, Number Sense and Operations | Measurement | Geometry and Spatial Sense |
|---|---|---|
| A. Represent and compare numbers less than 0 through familiar applications and extending the number line.<br><br>B. Compare, order and convert among fractions, decimals and percents.<br><br>C. Develop meaning for percents, including percents greater than 100 and less than 1.<br><br>D. Use models and pictures to relate concepts of ratio, proportion and percent.<br><br>E. Use order of operations, including use of parenthesis and exponents to solve multi-step problems, and verify and interpret the results.<br><br>F. Apply number system properties when performing computations.<br><br>G. Apply and explain the use of prime factorizations, common factors, and common multiples in problem situations. | A. Select appropriate units to measure angles, circumference, surface area, mass and volume, using:<br>• U.S. customary units; e.g., degrees, square feet, pounds, and other units as appropriate;<br>• metric units; e.g., square meters, kilograms and other units as appropriate.<br><br>B. Convert units of length, area, volume, mass and time within the same measurement system.<br><br>C. Identify appropriate tools and apply appropriate techniques for measuring angles, perimeter or circumference and area of triangles, quadrilaterals, circles and composite shapes, and surface area and volume of prisms and cylinders.<br><br>D. Select a tool and measure accurately to a specified level of precision.<br><br>E. Use problem solving techniques and technology as needed to solve problems involving length, weight, perimeter, area, volume, time and temperature. | A. Identify and label angle parts and the regions defined within the plane where the angle resides.<br><br>B. Draw circles, and identify and determine the relationships among the radius, diameter, center and circumference.<br><br>C. Specify locations and plot ordered pairs on a coordinate plane.<br><br>D. Identify, describe and classify types of line pairs, angles, two-dimensional figures and three-dimensional objects using their properties.<br><br>E. Use proportions to express relationships among corresponding parts of similar figures.<br><br>F. Describe and use the concepts of congruence, similarity and symmetry to solve problems. |

# K-12 Mathematics Benchmarks
By the end of the 5-7 program:

| Patterns, Functions and Algebra | Data Analysis and Probability | Mathematical Processes |
|---|---|---|
| A. Describe, extend and determine the rule for patterns and relationships occurring in numeric patterns, computation, geometry, graphs and other applications. | A. Read, create and use line graphs, histograms, circle graphs, box-and-whisker plots, stem-and-leaf plots, and other representations when appropriate. | A. Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives. |
| B. Represent, analyze and generalize a variety of patterns and functions with tables, graphs, words and symbolic rules. | B. Interpret data by looking for patterns and relationships, draw and justify conclusions, and answer related questions. | B. Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations. |
| C. Use variables to create and solve equations and inequalities representing problem situations. | C. Evaluate interpretations and conclusions as additional data are collected, modify conclusions and predictions, and justify new findings. | C. Use more than one strategy to solve a problem, and recognize there are advantages associated with various methods. |
| D. Use symbolic algebra to represent and explain mathematical relationships. | D. Compare increasingly complex displays of data, such as multiple sets of data on the same graph. | D. Recognize whether an estimate or an exact solution is appropriate for a given problem situation. |
| E. Use rules and variables to describe patterns, functions and other relationships. | E. Collect, organize, display and interpret data for a specific purpose or need. | C. Use deductive thinking to construct informal arguments to support reasoning and to justify solutions to problems. |
| F. Use representations, such as tables, graphs and equations, to model situations and to solve problems, especially those that involve linear relationships. | F. Determine and use the range, mean, median and mode to analyze and compare data, and explain what each indicates about the data. | F. Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures |
| G. Write, simplify and evaluate algebraic expressions. | G. Evaluate conjectures and predictions based upon data presented in tables and graphs, and identify misuses of statistical data and displays. | |

# K-12 Mathematics Benchmarks

By the end of the 5-7 program:

| Number, Number Sense and Operations | Measurement | Geometry and Spatial Sense |
|---|---|---|
| H. Use and analyze the steps in standard and non-standard algorithms for computing with fractions, decimals and integers.<br><br>I. Use a variety of strategies, including proportional reasoning, to estimate, compute, solve and explain solutions to problems involving integers, fractions, decimals and percents. | F. Analyze and explain what happens to area and perimeter or surface area and volume when the dimensions of an object are changed.<br><br>G. Understand and demonstrate the independence of perimeter and area for two-dimensional shapes and of surface area and volume for three-dimensional shapes. | H. Describe and use properties of triangles to solve problems involving angle measures and side lengths of right triangles.<br><br>H. Predict and describe results (size, position, orientation) of transformations of two-dimensional figures.<br><br>I. Identify and draw three-dimensional objects from different views (top, side, front and perspective).<br><br>J. Apply properties of equality and proportionality to solve problems involving congruent or similar figures; e.g., create a scale drawing. |

# K-12 Mathematics Benchmarks

By the end of the 5-7 program:

| Patterns, Functions and Algebra | Data Analysis and Probability | Mathematical Processes |
|---|---|---|
| I. Solve linear equations and inequalities symbolically, graphically and numerically.<br><br>I. Explain how inverse operations are used to solve linear equations.<br><br>J. Use formulas in problem-solving situations.<br><br>K. Graph linear equations and inequalities.<br><br>L. Analyze functional relationships, and explain how a change in one quantity results in a change in the other.<br><br>M. Approximate and interpret rates of change from graphical and numerical data. | H. Find all possible outcomes of simple experiments or problem situations, using methods such as lists, arrays and tree diagrams.<br><br>I. Describe the probability of an event using ratios, including fractional notation.<br><br>J. Compare experimental and theoretical results for a variety of simple experiments.<br><br>K. Make and justify predictions based on experimental and theoretical probabilities. | G. Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies.<br><br>H. Use representations to organize and communicate mathematical thinking and problem solutions.<br><br>I. Select, apply, and translate among mathematical representations to solve problems; e.g., representing a number as a fraction, decimal or percent as appropriate for a problem.<br><br>J. Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.<br><br>L. Recognize and use mathematical language and symbols when reading, writing and conversing with others. |

# Grade Five
## Number, Number Sense and Operations Standard

*Number and Number Systems*

1. Use models and visual representation to develop the concept of ratio as part-to-part and part-to-whole, and the concept of percent as part-to-whole.

2. Use various forms of "one" to demonstrate the equivalence of fractions; e.g., $\frac{18}{24} = \frac{9}{12} \times \frac{2}{2} = \frac{3}{4} \times \frac{6}{6}$ .

3. Identify and generate equivalent forms of fractions, decimals and percents.

4. Round decimals to the nearest hundredth and beyond and round fractions (including mixed numbers) to the nearest half.

5. Recognize and identify perfect squares and their roots.

*Meaning of Operations*

6. Represent and compare numbers less than 0 by extending the number line and using familiar applications; e.g., temperature, owing money.

7. Use commutative, associative, distributive, identity and inverse properties to simplify and perform computations.

8. Identify and use relationships between operations to solve problems.

9. Use order of operations, including use of parentheses, to simplify numerical expressions.

10. Justify through physical models, drawings, words, etc. why fractions need common denominators to be added or subtracted.

11. Explain how place value is related to addition and subtraction of decimals; e.g., $0.2 + 0.14$; the two tenths is added to the one tenth because they are both tenths.

*Computation and Estimation*

12. Use physical models, points of reference, and equivalent forms to add and subtract commonly used fractions with like and unlike denominators and decimals.

13. Estimate the results of computations involving whole numbers, fractions and decimals, using a variety of strategies.

## Measurement Standard

1. Identify and select appropriate units to measure angles; i.e., degrees.

2. Identify paths between points on a grid or coordinate plane and compare the lengths of the paths; e.g., shortest path, paths of equal length.

3. Demonstrate and describe the differences between covering the faces (surface area) and filling the interior (volume) of three-dimensional objects.

4. Demonstrate understanding of the differences among linear units (lengths), square units (area, surface area) and cubic units (volume).

5. Make conversions within the same measurement system while performing computations.

6. Use strategies to develop formulas for determining perimeter and area of triangles, rectangles and parallelograms, and volume of rectangular prisms.

7. Use benchmark angles (e.g.; 45º, 90º, 120º) to estimate the measure of angles, and use a tool to measure and draw angles.

## Geometry and Spatial Sense Standard

1. Draw circles, and identify and determine relationships among the radius, diameter, center and circumference; e.g., radius is half the diameter, the ratio of the circumference of a circle to its diameter is an approximation of $\pi$.

2. Use standard language to describe line, segment, ray, angle, skew, parallel and perpendicular.

3. Label vertex, rays, interior and exterior for an angle.

4. Describe and use properties of congruent figures to solve problems.

5. Use physical models to determine the sum of the interior angles of triangles and quadrilaterals.

6. Extend understanding of coordinate system to include points whose $x$ or $y$ values may be negative numbers.

7. Understand that the measure of an angle is determined by the degree of rotation of an angle side rather than the length of either side.

8. Predict what three-dimensional object will result from folding a two-dimensional net, then confirm the prediction by folding the net.

## Patterns, Functions and Algebra Standard

1.   Justify a general rule for a pattern or a function by using physical materials, visual representations, words, tables or graphs.

2. Use calculators or computers to develop patterns, and generalize them using tables and graphs.

*Use Algebraic Representations*

3. Use variables as unknown quantities in general rules when describing patterns and other relationships.

4. Create and interpret the meaning of equations and inequalities representing problem situations.

5. Model problems with physical materials and visual representations, and use models, graphs and tables to draw conclusions and make predictions.

*Analyze Change*

6. Describe how the quantitative change in a variable affects the value of a related variable; e.g., describe how the rate of growth varies over time, based upon data in a table or graph.

## Data Analysis and Probability Standard

*Data Collection*

1. Read, construct and interpret frequency tables, circle graphs and line graphs.

2. Select and use a graph that is appropriate for the type of data to be displayed; e.g., numerical vs. categorical data, discrete vs. continuous data.

3. Read and interpret increasingly complex displays of data, such as double bar graphs.

4. Determine appropriate data to be collected to answer questions posed by students or teacher, collect and display data, and clearly communicate findings.

*Statistical Methods*

5. Modify initial conclusions, propose and justify new interpretations and predictions as additional data are collected.

6. Determine and use the range, mean, median and mode, and explain what each does and does not indicate about the set of data.

*Probability*

7. List and explain all possible outcomes in a given situation.

8. Identify the probability of events within a simple experiment, such as three chances out of eight.

9. Use 0, 1 and ratios between 0 and 1 to represent the probability of outcomes for an event, and associate the ratio with the likelihood of the outcome.

10. Compare what should happen (theoretical/expected results) with what did happen (experimental/actual results) in a simple experiment.

11. Make predictions based on experimental and theoretical probabilities.

# Grade Six
## Number, Number Sense and Operations Standard

*Number and Number Systems*

1. Decompose and recompose whole numbers using factors and exponents (e.g., $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$ ), and explain why "squared" means "second power" and "cubed" means "third power."

2. Find and use the prime factorization of composite numbers. For example:

   a. Use the prime factorization to recognize the greatest common factor (GCF).
   b. Use the prime factorization to recognize the least common multiple (LCM).
   c. Apply the prime factorization to solve problems and explain solutions.

3. Explain why a number is referred to as being "rational," and recognize that the expression $\frac{a}{b}$ can mean $a$ parts of size $\frac{1}{b}$ each, $a$ divided by $b$, or the ratio of $a$ to $b$.

4. Describe what it means to find a specific percent of a number, using real-life examples.

5. Use models and pictures to relate concepts of ratio, proportion and percent, including percents less than 1 and greater than 100.

*Meaning of Operations*

6. Use the order of operations, including the use of exponents, decimals and rational numbers, to simplify numerical expressions.

7. Use simple expressions involving integers to represent and solve problems; e.g., if a running back loses 15 yards on the first carry but gains 8 yards on the second carry, what is the net gain/loss?

8. Represent multiplication and division situations involving fractions and decimals with models and visual representations; e.g., show with pattern blocks what it means to take $2\frac{2}{3} \div \frac{1}{6}$.

9. Give examples of how ratios are used to represent comparisons; e.g., part-to-part, part-to-whole, whole-to-part.

10. Recognize that a quotient may be larger than the dividend when the divisor is a fraction; e.g., $6 \div \frac{1}{2} = 12$.

*Computation and Estimation*

11. Perform fraction and decimal computations and justify their solutions; e.g., using manipulatives, diagrams, mathematical reasoning.

12. Develop and analyze algorithms for computing with fractions and decimals, and demonstrate fluency in their use.

13. Estimate reasonable solutions to problem situations involving fractions and decimals; e.g., $\frac{7}{8} + \frac{12}{13} \approx 2$ and $4.23 \times 5.8 \approx 25$.

14. Use proportional reasoning, ratios and percents to represent problem situations and determine the reasonableness of solutions.

15. Determine the percent of a number and solve related problems; e.g., find the percent markdown if the original price was $140, and the sale price is $100.


## Measurement Standard

*Measurement Units*

*Use Measurement Techniques and Tools*

1. Understand and describe the difference between surface area and volume.

2. Develop and use strategies and formulas for finding circumference and area of circles, and to determine the area of sectors; e.g., $\frac{1}{2}$ circle, $\frac{2}{3}$ circle, $\frac{1}{3}$ circle, $\frac{1}{4}$ circle.

3. Estimate perimeter or circumference and area for circles, triangles and quadrilaterals, and surface area and volume for prisms and cylinders by:

   a. estimating lengths using string or links, areas using tiles or grid, and volumes using cubes;
   b. measuring attributes (diameter, side lengths, or heights) and using established formulas for circles, triangles, rectangles, parallelograms and rectangular prisms.

4. Determine which measure (perimeter, area, surface area, volume) matches the context for a problem situation; e.g., perimeter is the context for fencing a garden, surface area is the context for painting a room.

5. Understand the difference between perimeter and area, and demonstrate that two shapes may have the same perimeter, but different areas or may have the same area, but different perimeters.

6. Describe what happens to the perimeter and area of a two-dimensional shape when the measurements of the shape are changed; e.g. length of sides are doubled.

## Geometry and Spatial Sense Standard

*Characteristics and Properties*

1. Classify and describe two-dimensional and three-dimensional geometric figures and objects by using their properties; e.g., interior angle measures, perpendicular/parallel sides, congruent angles/sides.

2. Use standard language to define geometric vocabulary: vertex, face, altitude, diagonal, isosceles, equilateral, acute, obtuse and other vocabulary as appropriate.

3. Use multiple classification criteria to classify triangles; e.g., right scalene triangle.

4. Identify and define relationships between planes; i.e., parallel, perpendicular and intersecting.

*Spatial Relationships*

5. Predict and describe sizes, positions and orientations of two-dimensional shapes after transformations such as reflections, rotations, translations and dilations.

*Transformations and Symmetry*

6. Draw similar figures that model proportional relationships; e.g., model similar figures with a 1 to 2 relationship by sketching two of the same figure, one with corresponding sides twice the length of the other.

*Visualization and Geometric Models*

7. Build three-dimensional objects with cubes, and sketch the two-dimensional representations of each side; i.e., projection sets.

## Patterns, Functions and Algebra Standard

*Use Patterns, Relations and Functions*

1. Represent and analyze patterns, rules and functions, using physical materials, tables and graphs.

2. Use words and symbols to describe numerical and geometric patterns, rules and functions.

*Use Algebraic Representations*

3. Recognize and generate equivalent forms of algebraic expressions, and explain how the commutative, associative and distributive properties can be used to generate equivalent forms; e.g., perimeter as $2(l + w)$ or $2l + 2w$.

4. Solve simple linear equations and inequalities using physical models, paper and pencil, tables and graphs.

5. Produce and interpret graphs that represent the relationship between two variables.

6. Evaluate simple expressions by replacing variables with given values, and use formulas in problem-solving situations.

*Analyze Change*

7. Identify and describe situations with constant or varying rates of change, and compare them.

8. Use technology to analyze change; e.g., use computer applications or graphing calculators to display and interpret rate of change.

## Data Analysis and Probability Standard

*Data Collection*

1. Read, construct and interpret line graphs, circle graphs and histograms.

2. Select, create and use graphical representations that are appropriate for the type of data collected.

3. Compare representations of the same data in different types of graphs, such as a bar graph and circle graph.

*Statistical Methods*

4. Understand the different information provided by measures of center (mean, mode and median) and measures of spread (range).

5. Describe the frequency distribution of a set of data, as shown in a histogram or frequency table, by general appearance or shape; e.g., number of modes, middle of data, level of symmetry, outliers.

*Probability*

6. Make logical inferences from statistical data.

7. Design an experiment to test a theoretical probability and explain how the results may vary.

# Grade Seven
## Number, Number Sense and Operations Standard

*Number and Number Systems*

1. Demonstrate an understanding of place value using powers of 10 and write large numbers in scientific notation.

2. Explain the meaning of exponents that are negative or 0.

3. Describe differences between rational and irrational numbers; e.g., use technology to show that some numbers (rational) can be expressed as terminating or repeating decimals and others (irrational) as non-terminating and non-repeating decimals.

*Meaning of Operations*

4. Use order of operations and properties to simplify numerical expressions involving integers, fractions and decimals.

5. Explain the meaning and effect of adding, subtracting, multiplying and dividing integers; e.g., how adding two integers can result in a lesser value.

*Computation and Estimation*

6. Simplify numerical expressions involving integers and use integers to solve real-life problems.

7. Solve problems using the appropriate form of a rational number (fraction, decimal or percent).

8. Develop and analyze algorithms for computing with percents and integers, and demonstrate fluency in their use.

10. Represent and solve problem situations that can be modeled by and solved using concepts of absolute value, exponents and square roots (for perfect squares).

## Measurement Standard

*Measurement Units*

1. Select appropriate units for measuring derived measurements; e.g., miles per hour, revolutions per minute.

2. Convert units of area, volume, length, mass and time within the same measurement system using proportional reasoning and a reference table when appropriate; e.g., square feet to square yards, ounces to pounds, and minutes to hours.

*Use Measurement Techniques and Tools*

3. Estimate a measurement to a greater degree of precision than the tool provides.

4. Solve problems involving proportional relationships and scale factors; e.g., scale models that require unit conversions within the same measurement system.

5. Analyze problem situations involving measurement concepts, select appropriate strategies, and use an organized approach to solve narrative and increasingly complex problems.

6. Use strategies to develop formulas for finding area of trapezoids and volume of cylinders and prisms.

7. Develop strategies to find the area of composite shapes using the areas of triangles, parallelograms, circles and sectors.

8. Understand the difference between surface area and volume and demonstrate that two objects may have the same surface area, but different volumes or may have the same volume, but different surface areas.

9. Describe what happens to the surface area and volume of a three-dimensional object when the measurements of the object are changed; e.g., length of sides are doubled.


# Geometry and Spatial Sense Standard


*Characteristics and Properties*

1. Use proportional reasoning to describe and express relationships between parts and attributes of similar and congruent figures.

2. Determine sufficient (not necessarily minimal) properties that define a specific two-dimensional figure or three-dimensional object. For example:

   a. Determine when one set of figures is a subset of another; e.g., all squares are rectangles.
   b. Develop a set of properties that eliminates all but the desired figure; e.g., only squares are quadrilaterals with all sides congruent and all angles congruent.

3. Use and demonstrate understanding of the properties of triangles. For example:

   a. Use Pythagorean Theorem to solve problems involving right triangles.
   b. Use triangle angle sum relationships to solve problems.

4. Determine necessary conditions for congruence of triangles.

5. Apply properties of congruent or similar triangles to solve problems involving missing lengths and angle measures.

*Spatial Relationships*

6. Determine and use scale factors for similar figures to solve problems using proportional reasoning.

*Transformations and Symmetry*

7. Identify the line and rotation symmetries of two-dimensional figures to solve problems.

8. Perform translations, reflections, rotations and dilations of two-dimensional figures using a variety of methods (paper folding, tracing, graph paper).

*Visualization and Geometric Models*

9. Draw representations of three-dimensional geometric objects from different views.

## Patterns, Functions and Algebra Standard

*Use Patterns, Relations and Functions*

1. Represent and analyze patterns, rules and functions with words, tables, graphs and simple variable expressions.

2. Generalize patterns by describing in words how to find the next term.

3. Recognize and explain when numerical patterns are linear or nonlinear progressions; e.g., 1, 3, 5, 7... is linear and 1, 3, 4, 8, 16... is nonlinear.

*Use Algebraic Representations*

4. Create visual representations of equation-solving processes that model the use of inverse operations.

5. Represent linear equations by plotting points in the coordinate plane.

6. Represent inequalities on a number line or a coordinate plane.

7. Justify that two forms of an algebraic expression are equivalent, and recognize when an expression is simplified; e.g., $4m = m + m + m + m$ or $a \cdot 5 + 4 = 5a + 4$.

8. Use formulas in problem-solving situations.

9. Recognize a variety of uses for variables; e.g., placeholder for an unknown quantity in an equation, generalization for a pattern, formula.

*Analyze Change*

10. Analyze linear and simple nonlinear relationships to explain how a change in one variable results in the change of another.

11. Use graphing calculators or computers to analyze change; e.g., distance-time relationships.

12 Use technology to analyze change; e.g., use computer applications or graphing calculators to display and interpret rate of change.

**Data Analysis and Probability Standard**

*Data Collection*    1. Read, create and interpret box-and-whisker plots, stem-and-leaf plots, and other types of graphs, when appropriate.

2. Analyze how decisions about graphing affect the graphical representation; e.g., scale, size of classes in a histogram, number of categories in a circle graph.

*Statistical Methods*    3. Analyze a set of data by using and comparing combinations of measures of center (mean, mode, median) and measures of spread (range, quartile, interquartile range), and describe how the inclusion or exclusion of outliers affects those measures.

4. Construct opposing arguments based on analysis of the same data, using different graphical representations.

5. Compare data from two or more samples to determine how sample selection can influence results.

6. Identify misuses of statistical data in articles, advertisements, and other media.

*Probability*    7. Compute probabilities of compound events; e.g., multiple coin tosses or multiple rolls of number cubes, using such methods as organized lists, tree diagrams and area models.

8. Make predictions based on theoretical probabilities, design and conduct an experiment to test the predictions, compare actual results to predicted results, and explain differences.

# K-12 Mathematics Benchmarks

By the end of the 8-10 program:

| Number, Number Sense and Operations | Measurement | Geometry and Spatial Sense |
|---|---|---|
| A. Use scientific notation to express large numbers and numbers less than one.<br><br>B. Identify subsets of the real number system.<br><br>C. Apply properties of operations and the real number system, and justify when they hold for a set of numbers.<br><br>D. Connect physical, verbal and symbolic representations of integers, rational numbers and irrational numbers.<br><br>E. Compare, order and determine equivalent forms of real numbers.<br><br>F. Explain the effects of operations on the magnitude of quantities.<br><br>*G. Estimate, compute and solve problems involving real numbers, including ratio, proportion and percent, and explain solutions.*<br><br>*H. Find the square root of perfect squares, and approximate the square root of non-perfect squares.* | A. Solve increasingly complex non-routine measurement problems and check for reasonableness of results.<br><br>B. Use formulas to find surface area and volume for specified three-dimensional objects accurate to a specified level of precision.<br><br>C. Apply indirect measurement techniques, tools and formulas, as appropriate, to find perimeter, circumference and area of circles, triangles, quadrilaterals and composite shapes, and to find volume of prisms, cylinders, and pyramids.<br><br>D. Use proportional reasoning and apply indirect measurement techniques, including right triangle trigonometry and properties of similar triangles, to solve problems involving measurements and rates.<br><br>E. Estimate and compute various attributes, including length, angle measure, area, surface area and volume, to a specified level of precision. | A. Formally define geometric figures.<br><br>B. Describe and apply the properties of similar and congruent figures; and justify conjectures involving similarity and congruence.<br><br>C. Recognize and apply angle relationships in situations involving intersecting lines, perpendicular lines and parallel lines.<br><br>D. Use coordinate geometry to represent and examine the properties of geometric figures.<br><br>E. Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools, such as straightedge, compass and technology.<br><br>F. Represent and model transformations in a coordinate plane and describe the results.<br><br>G. Prove or disprove conjectures and solve problems involving two- and three-dimensional objects represented within a coordinate system. |

# K-12 Mathematics Benchmarks

By the end of the 8-10 program:

| Patterns, Functions and Algebra | Data Analysis and Probability | Mathematical Processes |
|---|---|---|
| A. Generalize and explain patterns and sequences in order to find the next term and the *n*th term.<br><br>B. Identify and classify functions as linear or nonlinear, and contrast their properties using tables, graphs or equations.<br><br>C. Translate information from one representation (words, table, graph or equation) to another representation of a relation or function.<br><br>D. Use algebraic representations, such as tables, graphs, expressions, functions and inequalities, to model and solve problem situations.<br><br>E. Analyze and compare functions and their graphs using attributes, such as rates of change, intercepts and zeros.<br><br>*F. Solve and graph linear equations and inequalities.*<br><br>*G. Solve quadratic equations with real roots by graphing, formula and factoring.* | A. Create, interpret and use graphical displays and statistical measures to describe data; e.g., box-and-whisker plots, histograms, scatterplots, measures of center and variability.<br><br>B. Evaluate different graphical representations of the same data to determine which is the most appropriate representation for an identified purpose.<br><br>C. Compare the characteristics of the mean, median and mode for a given set of data, and explain which measure of center best represents the data.<br><br>D. Find, use and interpret measures of center and spread, such as mean and quartiles, and use those measures to compare and draw conclusions about sets of data.<br><br>E. Evaluate the validity of claims and predictions that are based on data by examining the appropriateness of the data collection and analysis. | A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution.<br><br>B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.<br><br>C Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the *x*-intercept of the graph of the function, apply proportional thinking when measuring, describing functions, and comparing probabilities.<br><br>D. Apply reasoning processes and skills to construct logical verifications or counter-examples to test conjectures and to justify and defend algorithms and solutions. |

# K-12 Mathematics Benchmarks

By the end of the 8-10 program:

| Number, Number Sense and Operations | Measurement | Geometry and Spatial Sense |
|---|---|---|
| *I. Estimate, compute and solve problems involving scientific notation, square roots and numbers with integer exponents.* | F. Write and solve real-world, multi-step problems involving money, elapsed time and temperature, and verify reasonableness of solutions. | G. Prove or disprove conjectures and solve problems involving two- and three-dimensional objects represented within a coordinate system.<br><br>H. Establish the validity of conjectures about geometric objects, their properties and relationships by counter-example, inductive and deductive reasoning, and critiquing arguments made by others.<br><br>J. Use right triangle trigonometric relationships to determine lengths and angle measures. |

# K-12 Mathematics Benchmarks

By the end of the 8-10 program:

| Patterns, Functions and Algebra | Data Analysis and Probability | Mathematical Processes |
|---|---|---|
| H. Solve systems of linear equations involving two variables graphically and symbolically.<br><br>I. Model and solve problem situations involving direct and inverse variation.<br><br>J. Describe and interpret rates of change from graphical and numerical data. | G. Construct convincing arguments based on analysis of data and interpretation of graphs.<br><br>G. Describe sampling methods and analyze the effects of method chosen on how well the resulting sample represents the population.<br><br>H. Use counting techniques, such as permutations and combinations, to determine the total number of options and possible outcomes.<br><br>I. Design an experiment to test a theoretical probability, and record and explain results.<br><br>J. Compute probabilities of compound events, independent events, and simple dependent events.<br><br>K. Make predictions based on theoretical probabilities and experimental results. | E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.<br><br>F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.<br><br>G. Write clearly and coherently about mathematical thinking and ideas.<br><br>H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner. |

# Grade Eight
## Number, Number Sense and Operations Standard

*Number and Number Systems*

1. Use scientific notation to express large numbers and small numbers between 0 and 1.

2. Recognize that natural numbers, whole numbers, integers, rational numbers and irrational numbers are subsets of the real number system.

*Meaning of Operations*

3. Apply order of operations to simplify expressions and perform computations involving integer exponents and radicals.

4. Explain and use the inverse and identity properties and use inverse relationships (addition/subtraction, multiplication/division, squaring/square roots) in problem solving situations.

*Computation and Estimation*

5. Determine when an estimate is sufficient and when an exact answer is needed in problem situations, and evaluate estimates in relation to actual answers; e.g., very close, less than, greater than.

6. Estimate, compute and solve problems involving rational numbers, including ratio, proportion and percent, and judge the reasonableness of solutions.

7. Find the square root of perfect squares, and approximate the square root of non-perfect squares as consecutive integers between which the root lies; e.g., $\sqrt{130}$ is between 11 and 12.

8. Add, subtract, multiply, divide and compare numbers written in scientific notation.

## Measurement Standard

*Measurement Units*

1. Compare and order the relative size of common U.S. customary units and metric units; e.g., mile and kilometer, gallon and liter, pound and kilogram.

2. Use proportional relationships and formulas to convert units from one measurement system to another; e.g., degrees Fahrenheit to degrees Celsius.

*Use Measurement Techniques and Tools*

3. Use appropriate levels of precision when calculating with measurements.

4. Derive formulas for surface area and volume and justify them using geometric models and common materials. For example, find:

   a. the surface area of a cylinder as a function of its height and radius;
   b. that the volume of a pyramid (or cone) is one-third of the volume of a prism (or cylinder) with the same base area and height.

5. Determine surface area for pyramids by analyzing their parts.

6. Solve and determine the reasonableness of the results for problems involving rates and derived measurements, such as velocity and density, using formulas, models and graphs.

7. Apply proportional reasoning to solve problems involving indirect measurements or rates.

8. Find the sum of the interior and exterior angles of regular convex polygons with and without measuring the angles with a protractor.

9. Demonstrate understanding of the concepts of perimeter, circumference and area by using established formulas for triangles, quadrilaterals, and circles to determine the surface area and volume of prisms, pyramids, cylinders, spheres and cones. (Note: Only volume should be calculated for spheres and cones.)

11. Use conventional formulas to find the surface area and volume of prisms, pyramids and cylinders and the volume of spheres and cones to a specified level of precision.


## Geometry and Spatial Sense Standard

*Characteristics and Properties*

1. Make and test conjectures about characteristics and properties (e.g., sides, angles, symmetry) of two-dimensional figures and three-dimensional objects.

2. Recognize the angles formed and the relationship between the angles when two lines intersect and when parallel lines are cut by a transversal.

3. Use proportions in several forms to solve problems involving similar figures (part-to-part, part-to-whole, corresponding sides between figures).

*Spatial Relationships*

4. Represent and analyze shapes using coordinate geometry; e.g., given three vertices and the type of quadrilateral, find the coordinates of the fourth vertex.

*Transformations and Symmetry*

5. Draw the results of translations, reflections, rotations and dilations of objects in the coordinate plane, and determine properties that remain fixed; e.g., lengths of sides remain the same under translations.

*Visualization and Geometric Models*

6. Draw nets for a variety of prisms, pyramids, cylinders and cones.

# Patterns, Functions and Algebra Standard

*Use Patterns, Relations and Functions*

1. Relate the various representations of a relationship; i.e., relate a table to graph, description and symbolic form.

2. Generalize patterns and sequences by describing how to find the *n*th term.

3. Identify functions as linear or nonlinear based on information given in a table, graph or equation.

*Use Algebraic Representations*

4. Extend the uses of variables to include covariants where *y* depends on *x*.

5. Use physical models to add and subtract monomials and polynomials, and to multiply a polynomial by a monomial.

6. Describe the relationship between the graph of a line and its equation, including being able to explain the meaning of slope as a constant rate of change and *y*-intercept in real-world problems.

7. Use symbolic algebra (equations and inequalities), graphs and tables to represent situations and solve problems.

8. Write, simplify and evaluate algebraic expressions (including formulas) to generalize situations and solve problems.

9. Solve linear equations and inequalities graphically, symbolically and using technology.

10. Solve 2 by 2 systems of linear equations graphically and by simple substitution.

11. Interpret the meaning of the solution of a 2 by 2 system of equations; i.e., point, line, no solution.

12. Solve simple quadratic equations graphically; e.g., $y = x^2 - 16$.

13. Compute and interpret slope, midpoint and distance given a set of ordered pairs.

*Analyze Change*

14. Differentiate and explain types of changes in mathematical relationships, such as linear vs. nonlinear, continuous vs. noncontinuous, direct variation vs. inverse variation.

15. Describe and compare how changes in an equation affects the related graphs; e.g., for a linear equation changing the coefficient of *x* affects the slope and changing the constant affects the intercepts.

16. *16. Use graphing calculators or computers to analyze change; e.g., interest compounded over time as a nonlinear growth pattern.*

## Data Analysis and Probability Standard

*Data Collection*

1. Use, create and interpret scatterplots and other types of graphs as appropriate.

2. Evaluate different graphical representations of the same data to determine which is the most appropriate representation for an identified purpose; e.g., line graph for change over time, circle graph for part-to-whole comparison, scatterplot for relationship between two variants.

3. Differentiate between discrete and continuous data and appropriate ways to represent each.

*Statistical Methods*

Compare two sets of data using measures of center (mean, mode, median) and measures of spread (range, quartiles, interquartile range, percentiles).

5. Explain the mean's sensitivity to extremes and its use in comparison with the median and mode.

6. Make conjectures about possible relationship in a scatterplot and approximate line of best fit.

7. Identify different ways of selecting samples, such as survey response, random sample, representative sample and convenience sample.

8. Describe how the relative size of a sample compared to the target population affects the validity of predictions.

9. Construct convincing arguments based on analysis of data and interpretation of graphs.

*Probability*

10. Calculate the number of possible outcomes for a situation, recognizing and accounting for when items may occur more than once or when order is important.

12. *Demonstrate an understanding that the probability of either of two disjoint events occurring can be found by adding the probabilities for each and that the probability of one independent event following another can be found by multiplying the probabilities.*

# Grade Nine
## Number, Number Sense and Operations Standard

*Number and*
*Number Systems*

1. Identify and justify whether properties (closure, identity, inverse, commutative and associative) hold for a given set and operations; e.g., even integers and multiplication.

2. Compare, order and determine equivalent forms for rational and irrational numbers.

*Meaning of*
*Operations*

3. Explain the effects of operations such as multiplication or division, and of computing powers and roots on the magnitude of quantities.

*Computation and*
*Estimation*

4. Demonstrate fluency in computations using real numbers.

5. Estimate the solutions for problem situations involving square and cube roots.

## Measurement Standard

*Measurement*
*Units*

1. Convert rates within the same measurement system; e.g., miles per hour to feet per second; kilometers per hour to meters per second.

*Use Measurement*
*Techniques and*
*Tools*

2. Use unit analysis to check computations involving measurement.

3. Use the ratio of lengths in similar two-dimensional figures or three-dimensional objects to calculate the ratio of their areas or volumes respectively.

4. Use scale drawings and right triangle trigonometry to solve problems that include unknown distances and angle measures.

5. Solve problems involving unit conversion for situations involving distances, areas, volumes and rates within the same measurement system.

## Geometry and Spatial Sense Standard

*Characteristics*
*and Properties*

1. Define the basic trigonometric ratios in right triangles: sine, cosine and tangent.

2. Apply proportions and right triangle trigonometric ratios to solve problems involving missing lengths and angle measures in similar figures.

*3. Analyze two-dimensional figures in a coordinate plane; e.g., use slope and distance*

*formulas to show that a quadrilateral is a parallelogram.*

## Patterns, Functions and Algebra Standard

*Use Patterns, Relations and Functions*

1. Define function with ordered pairs in which each domain element is assigned exactly one range element.

2. Generalize patterns using functions or relationships (linear, quadratic and exponential), and freely translate among tabular, graphical and symbolic representations.

3. Describe problem situations (linear, quadratic and exponential) by using tabular, graphical and symbolic representations.

4. Demonstrate the relationship among zeros of a function, roots of equations, and solutions of equations graphically and in words.

5. Describe and compare characteristics of the following families of functions: linear, quadratic and exponential functions; e.g., general shape, number of roots, domain, range, rate of change, maximum or minimum.

*Use Algebraic Representations*

6. Write and use equivalent forms of equations and inequalities in problem situations; e.g., changing a linear equation to the slope-intercept form.

7. Use formulas to solve problems involving exponential growth and decay.

8. Find linear equations that represent lines that pass through a given set of ordered pairs, and find linear equations that represent lines parallel or perpendicular to a given line through a specific point.

9. Solve and interpret the meaning of 2 by 2 systems of linear equations graphically, by substitution and by elimination, with and without technology.

10. Solve quadratic equations with real roots by factoring, graphing, using the quadratic formula and with technology.

11. Add, subtract, multiply and divide monomials and polynomials (division of polynomials by monomials only).

*Analyze Change*

12. Simplify rational expressions by eliminating common factors and applying properties of integer exponents.

13. Model and solve problems involving direct and inverse variation using proportional reasoning.

14. Describe the relationship between slope and the graph of a direct variation and inverse variation.

   *15. Describe how a change in the value of a constant in a linear or quadratic equation affects the related graphs.*


## Data Analysis and Probability Standard


*Data Collection*    1. Classify data as univariate (single variable) or bivariate (two variables) and as quantitative (measurement) or qualitative (categorical) data.

2. Create a scatterplot for a set of bivariate data, sketch the line of best fit, and interpret the slope of the line of best fit.

*Statistical*
*Methods*    3. Analyze and interpret frequency distributions based on spread, symmetry, skewness, clusters and outliers.

4. Describe and compare various types of studies (survey, observation, experiment), and identify possible misuses of statistical data.

5. Describe characteristics and limitations of sampling methods, and analyze the effects of random versus biased sampling; e.g., determine and justify whether the sample is likely to be representative of the population.

6. Make inferences about relationships in bivariant data, and recognize the difference between evidence of relationship (correlation) and causation.

*Probability*
7. Use counting techniques and the Fundamental Counting principle to determine the total number of possible outcomes for mathematical situations.

8. Describe, create and analyze a sample space and use it to calculate probability.

9. Identify situations involving independent and dependent events, and explain differences between, and common misconceptions about, probabilities associated with those events.

   *10. Use theoretical and experimental probability, including simulations or random numbers, to estimate probabilities and to solve problems dealing with uncertainty; e.g., compound events, independent events, simple dependent events.*

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# Grade Ten
## Number, Number Sense and Operations Standard

*Number and*
*Number Systems*

1. Connect physical, verbal and symbolic representations of irrational numbers; e.g., construct $\sqrt{2}$ as a hypotenuse or on a number line.

*Meaning of*
*Operations*

2. Explain the meaning of the *n*th root.

*Computation and*
*Estimation*

3. Use factorial notation and computations to represent and solve problem situations involving arrangements.

4. *Approximate the* n*th root of a given number greater than zero between consecutive integers when* n *is an integer; e.g., the 4th root of 50 is between 2 and 3.*

## Measurement Standard

*Use Measurement*
*Techniques and*
*Tools*

1. Explain how a small error in measurement may lead to a large error in calculated results.

2. Calculate relative error.

3. Explain the difference between absolute error and relative error in measurement.

4. Give examples of how the same absolute error can be problematic in one situation but not in another; e.g., compare "accurate to the nearest foot" when measuring the height of a person versus when measuring the height of a mountain.

5. *Determine the measures of central and inscribed angles and their associated major and minor arcs.*

## Geometry and Spatial Sense Standard

*Characteristics*
*and Properties*

1. Formally define and explain key aspects of geometric figures, including:

   a. interior and exterior angles of polygons;
   b. segments related to triangles (median, altitude, midsegment);
   c. points of concurrency related to triangles (centroid, incenter, orthocenter, circumcenter);
   d. circles (radius, diameter, chord, circumference, major arc, minor arc, sector, segment, inscribed angle).

*Spatial*

2. Recognize and explain the necessity for certain terms to remain undefined,

*Relationships*      such as point, line and plane.

3. Make, test and establish the validity of conjectures about geometric properties and relationships using counterexample, inductive and deductive reasoning, and paragraph or two-column proof, including:

     a. prove the Pythagorean Theorem;
     b. prove theorems involving triangle similarity and congruence;
     c. prove theorems involving properties of lines, angles, triangles and quadrilaterals;
     d. test a conjecture using basic constructions made with a compass and straightedge or technology.

4. Construct right triangles, equilateral triangles, parallelograms, trapezoids, rectangles, rhombuses, squares and kites, using compass and straightedge or dynamic geometry software.

5. Construct congruent figures and similar figures using tools, such as compass, straightedge, and protractor or dynamic geometry software.

*Transformation and Symmetry*    6. Identify the reflection and rotation symmetries of two- and three-dimensional figures.

7. Perform reflections and rotations using compass and straightedge constructions and dynamic geometry software.

8. Derive coordinate rules for translations, reflections and rotations of geometric figures in the coordinate plane.

9. Show and describe the results of combinations of translations, reflections and rotations (compositions); e.g., perform compositions and specify the result of a composition as the outcome of a single motion, when applicable.

*Visualization and Geometric Models*    10. *Solve problems involving chords, radii and arcs within the same circle.*

## Patterns, Functions and Algebra Standard

*Use Patterns, Relations and Functions*    1. Define function formally and with f(*x*) notation.

2. Describe and compare characteristics of the following families of functions: square root, cubic, absolute value and basic trigonometric functions; e.g., general shape, possible number of roots, domain and range.

*Use Algebraic Representations*    3. Solve equations and formulas for a specified variable; e.g., express the base of a triangle in terms of the area and height.

4. Use algebraic representations and functions to describe and generalize

geometric properties and relationships.

5. Solve simple linear and nonlinear equations and inequalities having square roots as coefficients and solutions.

6. Solve equations and inequalities having rational expressions as coefficients and solutions.

7. Solve systems of linear inequalities.

8. Graph the quadratic relationship that defines circles.

9. Recognize and explain that the slopes of parallel lines are equal and the slopes of perpendicular lines are negative reciprocals.

10. Solve real-world problems that can be modeled using linear, quadratic, exponential or square root functions.

11. Solve real-world problems that can be modeled, using systems of linear equations and inequalities.

*Analyze Change*

12. *Describe the relationship between slope of a line through the origin and the tangent function of the angle created by the line and the positive* x*-axis.*


## Data Analysis and Probability Standard


*Data Collection*

1. Describe measures of center and the range verbally, graphically and algebraically.

2. Represent and analyze bivariate data using appropriate graphical displays (scatterplots, parallel box-and-whisker plots, histograms with more than one set of data, tables, charts, spreadsheets) with and without technology.

3. Display bivariate data where at least one variable is categorical.

4. Identify outliers on a data display; e.g., use interquartile range to identify outliers on a box-and-whisker plot.

*Statistical Methods*

5. Provide examples and explain how a statistic may or may not be an attribute of the entire population; e.g., intentional or unintentional bias may be present.

6. Interpret the relationship between two variables using multiple graphical displays and statistical measures; e.g., scatterplots, parallel box-and-whisker plots, and measures of center and spread.

*Probability*

7. Model problems dealing with uncertainty with area models (geometric

probability).

8. Differentiate and explain the relationship between the probability of an event and the odds of an event, and compute one given the other.

# K-12 Mathematics Benchmarks

By the end of the 11-12 program:

| Number, Number Sense and Operations | Measurement | Geometry and Spatial Sense |
|---|---|---|
| A. Demonstrate that vectors and matrices are systems having some of the same properties of the real number system.<br><br>B. Develop an understanding of properties of and representations for addition and multiplication of vectors and matrices.<br><br>C. Apply factorials and exponents, including fractional exponents, to solve practical problems.<br><br>D. Demonstrate fluency in operations with real numbers, vectors and matrices, using mental computation or paper and pencil calculations for simple cases and technology for more complicated cases.<br><br>E. Represent and compute with complex numbers. | A. Explain differences among accuracy, precision and error, and describe how each of those can affect solutions in measurement situations.<br><br>B. Apply various measurement scales to describe phenomena and solve problems.<br><br>C. Estimate and compute areas and volume in increasingly complex problem situations.<br><br>D. Solve problem situations involving derived measurements; e.g., density, acceleration. | Use trigonometric relationships to verify and determine solutions in problem situations.<br><br>Represent transformations within a coordinate system using vectors and matrices. |

# K-12 Mathematics Benchmarks

By the end of the 11-12 program:

| Patterns, Functions and Algebra | Data Analysis and Probability | Mathematical Processes |
|---|---|---|
| A. Analyze functions by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.<br><br>B. Use the quadratic formula to solve quadratic equations that have complex roots.<br><br>C. Use recursive functions to model and solve problems; e.g., home mortgages, annuities.<br><br>D. Apply algebraic methods to represent and generalize problem situations involving vectors and matrices. | A. Create and analyze tabular and graphical displays of data using appropriate tools, including spreadsheets and graphing calculators.<br><br>B. Use descriptive statistics to analyze and summarize data, including measures of center, dispersion, correlation and variability.<br><br>C. Design and perform a statistical experiment, simulation or study; collect and interpret data; and use descriptive statistics to communicate and support predictions and conclusions.<br><br>D. Connect statistical techniques to applications in workplace and consumer situations. | A. Construct algorithms for multi-step and non-routine problems.<br><br>B. Construct logical verifications or counter-examples to test conjectures and to justify or refute algorithms and solutions to problems.<br><br>C. Assess the adequacy and reliability of information available to solve a problem.<br><br>D. Select and use various types of reasoning and methods of proof.<br><br>E. Evaluate a mathematical argument and use reasoning and logic to judge its validity.<br><br>F. Present complete and convincing arguments and justifications, using inductive and deductive reasoning, adapted to be effective for various audiences.<br><br>    Understand the difference between a statement that is verified by mathematical proof, such as a theorem, and one that is verified empirically using examples or data. |

# K-12 Mathematics Benchmarks

By the end of the 11-12 program:

| Patterns, Functions and Algebra | Data Analysis and Probability | Mathematical Processes |
| --- | --- | --- |
| | | Use formal mathematical language and notation to represent ideas, to demonstrate relationships within and among representation systems, and to formulate generalizations.<br><br>I.  Communicate mathematical ideas orally and in writing with a clear purpose and appropriate for a specific audience.<br><br>J.  Apply mathematical modeling to workplace and consumer situations, including problem formulation, identification of a mathematical model, interpretation of solution within the model, and validation to original problem situation. |

# Algebra I

**Standard:    Patterns, Functions & Algebra**

The student will demonstrate the ability to:

1) Model real data using functions.
2) Analyze zeros and intercepts of functions.
3) Translate among tables, expression and graphs.
4) Use a graphing calculator to generate graphs of functions.
5) Describe characteristics of polynomial functions and use in problem solving situations.
6) Describe problem situations by using and relating numerical, symbolic and graphical representations.
7) Use the language and notation of functions in symbolic and graphic settings.
8) Recognize, relate and use the equivalent ideas of zeros of a function, tools of an equation, and solution of an equation in terms of graphical and symbolic representations.
9) Describe and use the logic of equivalence in working with equations, inequalities, and functions.
10) Develop graphical techniques of solution for problem situations involving functions.
11) Make arguments and proofs in algebraic settings.
12) Factor the difference of two squares.
13) Determine slope, midpoint, and distance.
14) Explore and combine rational functions.
15) Explore factoring techniques.
16) Solve quadratic equations by factoring and formula.
17) Set-up and solve linear equations.
18) Solve systems of linear equations with two variables.

**Standard:    Measurement**

The student will demonstrate the ability to:

1) Use measurements.
2) Measure accurately to a specific designation.
3) Apply measurements to real world experiments.
4) Establish ratios and rates.
5) Construct and interpret maps, tables, charts, and graphs.
6) Use right triangle measures.
7) Determine perimeter area and volume.

**Standard:      Number, Number Sense & Operations**

The student will demonstrate ability to:

1) Use estimation in problem solving to transform the conditions of the problem to explore solutions.
2) Interpret shapes of graphs.
3) Validate conclusions.

**Standard:      Geometry and Spatial Sense**

The student will demonstrate the ability to:

1) Define the basic trigonometric ratios in right triangles: sine, cosine and tangent.
2) Apply proportions and right triangle trigonometric ratios to solve problems involving missing lengths and angle measures in similar figures.
3) Analyze two-dimensional figures in a coordinate plane; e.g., use slope and distance formulas to show that a quadrilateral is a parallelogram.

**Standard:      Data Analysis and Probability**

The student will demonstrate the ability to:

1) Organize data into tables, charts, and graphs.
2) Understand and apply measures of central tendency.
3) Predict from data.

# Geometry

**Standard:      Patterns, Functions & Algebra**

The Student will demonstrate the ability to:

1) Interpret graphs.
2) Identify conic sections.
3) Apply trigonometric functions to problems involving triangles.
4) Use function notation to describe geometric situations.
5) Apply the algebra skills of solving equations and inequalities factoring, determining slope midpoint and distance.
6) Develop linear equations and solve systems of equations.
7) Use and apply measures of central tendency.
8) Use coordinate arguments in geometric proofs.
9) Symbolize transformations of figures and graphs.
10) Analyze and apply the functions of trigonometry.

**Standard:      Measurement**

The student will demonstrate the ability to:

1) Measure angles and segments.
2) Determine perimeter, area, and volume.
3) Establish rations and solve rate change problems.
4) Apply Pythagorean theorem to real life measurement situations.

**Standard:      Number, Number Sense & Operations**

The student will demonstrate the ability to:

1) Compare, order, and determine equivalence of real numbers.
2) Compute and solve problems involving real numbers.
3) Compare and contrast the real number system, rational number system, irrational number system and whole number system.
4) Transform problem situations using estimation.
5) Interpret shapes of graphs.
6) Validate conclusions.

**Standard:      Geometry and Spatial Sense**

The student will demonstrate the ability to:

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1) Create and interpret drawings of three dimensional objects.
2) Represent problem situations with geometric models and apply properties of figures.
3) Describe and apply the properties of similar and congruent figures.
4) Apply the Pythagorean theorem to given situations.
5) Demonstrate an understanding of angles and parallel and perpendicular lines.

**Standard:** **Data Analysis and Probability**

The student will demonstrate the ability to:

1) Organize data into tables, charts and graphs.
2) Understand and apply measures of central tendency.
3) Predict from data.

# Algebra II

**Standard:** **Patterns, Functions & Algebra**

The Student will demonstrate the ability to:

1) Analyze graphs and equations of conic sections.
2) Analyze the effects of parameter changes on graphs.
3) Use a graphing calculator to graph functions.
4) Determine the connections among exponential functions, log functions, complex numbers, and series.
5) Explore graphs in three dimensions and functions of several variables.
6) Describe problem situations by using and relating numerical, symbolic and graphical representations.
7) Recognize, relate and use the equivalent ideas of zeros of a function, root of an equation, and solution of an equation in terms of graphical and symbolic representations.
8) Describe and use the logic of equivalence in working with equations, inequalities, and functions.
9) Develop graphical techniques of solution for problem situations involving functions.
10) Make arguments and proofs in algebraic settings.

**Standard:** **Measurement**

The student will demonstrate the ability to:

1) Understand and solve rate and change problems.
2) Use Pythagorean theorem in distance problems.
3) Compute rates of growth or decay.
4) Compute and interpret real problems involving annuities.
5) Determine area and volume.

**Standard:** **Number, Number Sense & Operations**

The student will demonstrate the ability to:

1) Compare, order and determine equivalence of complex numbers.
2) Compute and solve problems involving complex numbers.
3) Compare and contrast the various subgroups in the complex numbers system.
4) Use the operations of the complex number system.
5) Use estimation to determine reasonableness of problem situations in a wide variety of applications.

6) Estimate the shapes of graphs of various functions and algebraic expressions.

**Standard:     Geometry and Spatial Sense**

The student will demonstrate the ability to:

1) Analyze functions and relations in 2 or 3 dimensional geometry.
2) Use the geometric properties of plane and solid figures in solving algebraic problems.
3) Apply the Pythagorean theorem to given situations.
4) Deduce properties of figures using vectors and transformations.

**Standard:     Data Analysis and Probability**

The student will demonstrate the ability to:

1) Understand and apply measures of central tendency, variability and correlations.
2) Predict, given data, using curve fitting.
3) Use experimental or theoretical probability.
4) Read, interpret and use statistical presentations to draw conclusions and make predictions.
5) Determine probabilities of events.
6) Use and interpret the normal curve.

# Pre Calculus/Advanced Math

**Standard:** **Patterns, Functions & Algebra**

The student will demonstrate the ability to:

1) Use right triangle relationship and Law of Sines and Law of Cosines.
2) Connect algebraic definitions of conics with type and special properties.
3) Use graphing calculator to assist in graphing functions and demonstrate effects of parameters.
4) Use rational, transcendented, circular, trigonometric, exponential and logarithmic functions.
5) Model real world phenomena with given functions.
6) Graph using polar coordinates.
7) Explore functions and graphs of several variables.
8) Develop and use vectors, polar and parametric equations, recursive functions, matrices, sequences and series, and inverse functions in problem solving.
9) Develop limits and illustrate in sequences and other functions.
10) Explore proofs in mathematical induction.

**Standard:** **Measurement**

The student will demonstrate the ability to:

1) Apply appropriate measures in solving problems using exponential, logarithmic and trigonometric functions.

**Standard:** **Number, Number Sense & Operations**

The student will demonstrate ability to:

1) Extend all number relations to the complex number system.
2) Use estimation to analyze reasonableness of solutions.
3) Estimate effects of parameters on graphs.
4) Estimate zeros of functions.

**Standard:** **Geometry and Spatial Sense**

The student will demonstrate the ability to:

1) Apply transformation and coordinate geometry in problem solving.

2) Use vectors to deduce properties of geometric figures.
3) Apply vectors to Euclidian transformation and problem solving.

**Standard:     Data Analysis and Probability**

The student will demonstrate the ability to:

1) Use statistical experiment to analyze and determine the outcome of a problem.
2) Understand and apply the properties of the normal curve.
3) Use the concept of a random variable.
4) Create and interpret discrete probability distributions.

# Integrated Math I
9<sup>th</sup> Grade

**Standard:**    **Number, Number Sense & Operations**

The student will demonstrate the ability to:

1) Model real-world phenomena with phenomena with polynomial and exponential functions.
2) Explore the relationship between zeros and intercepts of function.
3) Translate among tables, algebraic expressions, and graphs of functions.
4) Use a graphing calculator or computer to generate the graph of a function.
5) Explore the relation between a linear function and its inverse.
6) Describe the general characteristics of polynomial functions and use them in problem-solving situations.

**Standard:**    **Geometry and Spatial Sense**

The student will demonstrate the ability to:

1) Create and interpret drawings of three-dimensional objects.
2) Represent problem situations with geometric models and apply properties of figures.
3) Describe and apply the properties of similar and congruent figures.
4) Apply the Pythagorean theorem.
5) Demonstrate an understanding of angles and parallel and perpendicular lines.

**Standard:**    **Patterns, Functions & Algebra**

The student will demonstrate the ability to:

1) Describe problem situations by using and relating numerical, symbolic, and graphical representations.
2) Use the language and notation of functions in symbolic and graphing settings.
3) Recognize, relate, and use the equivalent ideas of zeros of a function, roots of an equation, and solution of an equation in terms of graphical and symbolic representations.
4) Describe and use the logic of equivalence in working with equations, inequalities, and functions.

5) Develop graphical techniques of solution for problem situations involving functions.
6) Explore and describe characterizing features of functions.
7) Make arguments and proofs in algebraic settings.
8) Factor the difference of two squares.
9) Determine slope, midpoint, and distance.
10) Explore and combine rational functions.
11) Explore factoring technique.
12) Solve quadratic equations by factoring and formula.
13) Set up and solve linear equations.
14) Solve systems of linear equations with two variables.

## Standard:    Measurement

The student will demonstrate the ability to:

1) Use measurements.
2) Measure accurately to a specific designation.
3) Apply measurements to real world experiments.
4) Establish ratios and rates.
5) Construct and interpret maps, tables, charts, and graphs.
6) Use right triangle measures.
7) Determine perimeter area and volume.

## Standard:    Data Analysis & Probability

The student will demonstrate the ability to:

1) Organize data into tables, charts, and graphs.
2) Understand and apply measures of central tendency, variability, and correlation.
3) Transform data to aid in data interpretation and prediction.

# Integrated Math II
Grade 10

**Standard:     Number, Number Sense & Operations**

The student will demonstrate the ability to:

1) Explore the conic sections and graph using a graphing calculator or computer.
2) Apply trigonometric functions to problem situations involving triangles.
3) Discover general relationships between the algebraic description of a conic, the kind of conic, and the special properties of that conic.

**Standard:     Geometry and Spatial Sense**

The student will demonstrate the ability to:

1) Explore inductive and deductive reasoning through applications to various subject areas.
2) Translate between synthetic and coordinate representations.
3) Identify congruent and similar figures using transformations with computer programs.
4) Deduce properties of figures using transformations and using coordinates.
5) Use deductive reasoning.
6) Explore compass and straight-edge constructions in the context of geometric theorems.
7) Demonstrate an understanding of and ability to use proof.
8) Use a variety of techniques of proof, such as synthetic, transformational, and coordinate proofs.
9) Use a variety of proof formats, including the T- (or two-column) proof and the paragraph proof.
10) Investigate different proofs of theorems.
11) Develop an understanding of an axiomatic system.

**Standard: Patterns, Functions & Algebra**

The student will demonstrate the ability to:

1) Describe geometric situations and phenomena using variables, equations and functions.
2) Describe measures of central tendency, mean, median, mode, and variance, algebraically and graphically.
3) Represent inequalities on the number line and in the coordinate plane.
4) Use coordinate arguments in making geometric proofs.
5) Symbolize transformations of figures and graphs.

6) Explore the geometric basis for the functions of trigonometry.
7) Graph linear functions.

**Standard:    Measurement**

The student will demonstrate the ability to:

1) Measure angles and segments.
2) Determine perimeter, area, and volume.
3) Establish rations and solve rate change problems.
4) Apply Pythagorean theorem to real life measurement situations.

**Standard:    Data Analysis & Probability**

The student will demonstrate the ability to:

1) Use curve fitting to predict from data.
2) Use experimental or theoretical probability, as appropriate, to represent and solve problems involving uncertainty.
3) Use computer simulations and random number generators to estimate probabilities.
4) Test hypotheses using appropriate statistics
5) Read, interpret, and use tables, charts, and graphs to identify patterns, note trends, draw conclusions, and make predictions.
**6)** Determine probabilities of events involving unbiased objects.

# Integrated Math 3
11<sup>th</sup> Grade

**Standard:      Number, Number Sense & Operations**

The student will demonstrate the ability to:

1)  Explore periodic real-world phenomena using the sine and cosine functions.
2)  Analyze the effects of parameter changes on graphs.
3)  Use a graphing calculator or computer to graph functions
4)  Develop an understanding of rational and transcendental functions.
5)  Understand the connection between trigonometric and circular functions.
6)  Use circular functions to model periodic real-world functions.
7)  Solve trigonometric equations and verify trigonometric identities graphically and analytically.
8)  Understand the connections between trigonometric functions and polar coordinates, exponential functions, logarithmic functions, complex numbers, and series.

**Standard:      Geometry and Spatial Sense**

The student will demonstrate the ability to:

1)  Apply transformations and coordinates in problem solving.
2)  Represent problem situations with geometric models and apply properties of figures deduce properties of figures using vectors.
3)  Analyze properties of Euclidian transformations and relate translations to vectors.
4)  Apply vectors in problem solving.
5)  Further develop understanding of axiomatic systems by investigating and comparing various geometries.

**Standard:      Patterns, Functions & Algebra**

The student will demonstrate the ability to:

1)  Develop and use vectors to represent direction and magnitude including operations.
2)  Use polar and parametric equations to describe, graph, and solve problem situations.
3)  Represent sequences and series as functions both algebraically and graphically.
4)  Explore recursive functions and procedures using spread sheets and other computer utilities and use notations appropriate to these problem situations.

5) Describe and solve algebraic situations with matrice.
6) Describe and use the inverse relationship between functions, including exponential and logarithmic.
7) Analyze and describe the errors and sources of errors that can be made when using computers and calculators to solve problems.
8) Decide when a problem situation is best solved using a computer, calculator, paper and pencil, or mental arithmetic/estimation techniques.
9) Explore relationships between complex numbers and vectors.
10) Make arguments concerning limits, convergence and divergence in contexts involving sequences, series, and other types of function.
11) Represent transformations in the plane with matrices.
12) Contrast and compare the algebras of the rational, real, and complex numbers with characteristics of a matrix algebra system.
13) Construct polynomial approximations of a function over specified intervals of convergence.
14) Examine complex numbers as zeros of functions.
15) Translate verbal statements into symbolic language.
16) Simplify algebraic expressions.
17) Use laws and exponents (including scientific notation).

**Standard:     Measurement**

The student will demonstrate the ability to:

1) Explain and apply measurements in a variety of problem situations.
2) Understand and solve rate change problems.
3) Comprehend and compute rates of growth or decay.
4) Comprehend, compute and interpret real problems.
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**Standard:     Data Analysis & Probability**

The student will demonstrate the ability to:

1) Use sampling and recognize its role in statistical claims.
2) Design a statistical experiment to study a problem, conduct the experiment, and interpret and communicate the outcomes.
3) Describe, in general terms, the normal curve and use its properties.

# Integrated Math 4
12<sup>th</sup> Grade

**Standard:**     **Number, Number Sense & Operations**

The student will demonstrate the ability to:

1) Model real-world phenomena with a variety of functions.
2) Graph using polar coordinates.
3) Explore graphs in three dimensions.
4) Explore functions of several variables.
5) Explore recursive functions using spreadsheets and/or programming languages.

**Standard:**     **Geometry and Spatial Sense**

Selected review objectives from grades 9-11.

**Standard:**     **Patterns, Functions & Algebra**

The student will demonstrate the ability to:

1) Expand and extend the idea of vectors and linear algebra to higher dimensional situations.
2) Use the idea of independent basis elements for a vector space and associated fundamental concepts of finite dimensional linear algebra.
3) Develop and communicate arguments about limit situations.
4) Use matrices to describe and apply transformations.
5) Develop and use polar and parametric equations to represent problem situations.
6) Explore proofs by mathematical induction.

**Standard:**     **Measurement**

Selected objectives as appropriate from grades 9-11.

**Standard: Data Analysis & Probability**

The student will demonstrate the ability to:

1) Create and interpret discrete probability distributions.
2) Understand the concept of random variable.
3) Apply the concept of a random variable to generate in interpret probability distributions, including binomial, uniform, normal, and chi square.

# Contemporary College Math

The student will demonstrate the ability to:

1) Understand and use the techniques of combinatorial analysis.
2) Use the Binomial theorem.
3) Analyze and compute with statistical data.
4) Use the graphing calculator effecting in data analysis.
5) Solve systems of equations using a variety of methods.
6) Solve and graph using polar coordinates.
7) Solve parametric equations.
8) Graph algebraic curves.
9) Understand functions and limits and solve problems using Limit Theory.
10) Perform operations of Differential Calculus.
11) Perform operations of Integral Calculus.

# Advanced Placement Calculus
(Course Content and Indicators Determined by National Advanced Placement Program)

The student will demonstrate the ability to:

1) Work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.
2) Understand the meaning of the derivative in terms of a rate of change and local linear approximation and they should be able to use derivatives to solve a variety of problems.
3) Understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of a rate of change and should be able to use integrals to solve a variety of problems.
4) Understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
5) Communicate mathematics both orally and in well-written sentences and should be able to explain solutions to problems.
6) Model a written description of a physical situation with a function, a differential equation, or an integral.
7) Use technology to help solve problems, experiment, interpret results, and verify conclusions.
8) Determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
9) Develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.